Pre-Calculus Chapter 2 Pre-Test

1.) (2.5 pts each, 5 pts total) Determine whether each of the following is a polynomial. If so, identify the degree

a) $f(x) = 2x^5 - 3x^3 + 7x^2 - 9x$

exponents must be whole numbers – no fractions, no negatives

polynomial

5th degree

b) $f(x) = 5x^3 + 12x^2 + \sqrt{9x}$

$x^{\frac{1}{2}}$

no

2.) (5 pts) Graph the quadratic function, which is given in standard form

$f(x) = (x + 2)^2 - 4$

vertex

vertex $(-2,-4)$

y-int

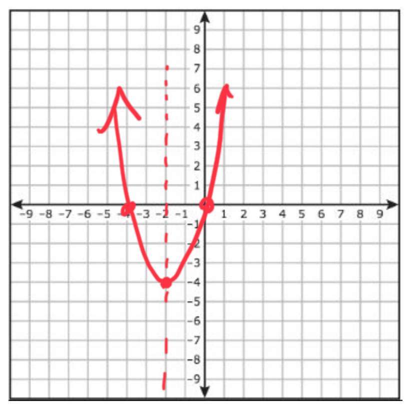
$x = 0$

$(x+2)^2 - 4$

$(0+2)^2 - 4$

$4 - 4 = 0$

$\{(0,0)\}$



3.) (10 pts) Rewrite the quadratic function in standard form by completing the square. Then graph.

$f(x) = (2x^2 + 8x) + 5$

y-int

1.) zero it

2.) factor "a"

3.) $\left(\frac{b}{2}\right)^2$

$\left(\frac{4}{2}\right)^2 = 4$

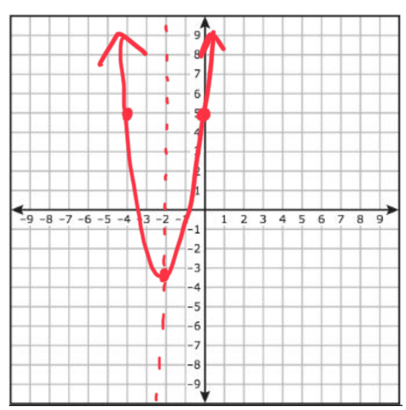
$\left(\frac{2x^2}{2} + \frac{8x}{2}\right) + 5$

$2(x^2 + 4x) + 5$

$+4$ $\quad$ $2(-4)$

$2(x^2 + 4x + 4) + 5 - 8$

$\boxed{2(x+2)^2 - 3 = y}$



4.) (5 pts) Find all of the real zeros (and their state of multiplicities) for the polynomial.

$$f(x) = 6x^2(x - 2)^4(x + 7)^3$$

$\frac{6x}{6} = \frac{0}{6}$

$x = 0$

$x - 2 = 0$
$+2 \quad +2$
$x = 2$

$x = 0$ mult: 2 $\quad$ $x = 2$ mult: 4 $\quad$ $x = -7$ mult: 3

5.) (10 pts) Find a polynomial of minimum degree that has the given zeros.

-2, 0, 1, 3

$x = -2$
$+2 \quad +2$
$x + 2 = 0$

$$\boxed{(x+2)(x)(x-1)(x-3)}$$

6.) (10 pts) For the polynomial function: (a) list each real zero and its multiplicity; (b) determine whether the graph touches or crosses at each x-intercept; (c) find the y-intercept; (d) sketch-ish the graph.

$$f(x) = x^5 - 4x^3$$

a) $x^5 - 4x^3$

$x^3(x^2 - 4)$

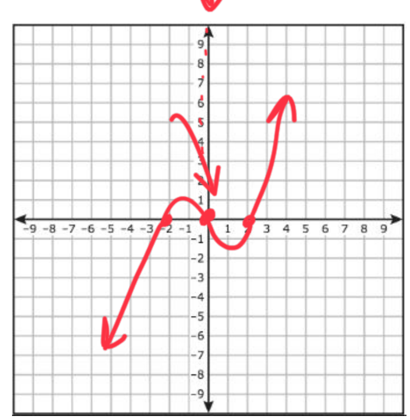
$x^3(x+2)(x-2)$

$x = 0$ mult: 3 $\quad$ $x = -2$ mult: 1 $\quad$ $x = 2$ mult: 1

$x = 0$

$(0)^5 - 4(0)^3$

$0 - 0 = 0$



$1: (1)^3(1+2)(1-2)$
$+ \quad + \quad -$
$= -$

$3: (3)^3(3+2)(3-2)$
$+ \quad + \quad +$
$= +$

$x^3(x+2)(x-2)$

$-4: \quad (-4)^3(-4+2)(-4-2) = -$
$- \quad - \quad -$

$-1 \quad (-1)^3(-1+2)(-1-2) = +$
$- \quad + \quad -$

7.) (7.5 pts each, 15 pts total) Divide the polynomials by either long division or synthetic division.

a) $(x^4 - 2x^3 - 7x^2 + 8x + 12) \div (x + 2)$

$$
\begin{array}{r}
x^3 - 4x^2 + x + 6 \\
x+2 \,\big)\, x^4 - 2x^3 - 7x^2 + 8x + 12 \\
-x^4 - 2x^3 \qquad\qquad\qquad \\
\hline
-4x^3 - 7x^2 \qquad\quad \\
+4x^3 + 8x^2 \qquad\quad \\
\hline
x^2 + 8x \qquad \\
-x^2 - 2x \qquad \\
\hline
6x + 12 \\
-6x - 12 \\
\hline
0
\end{array}
$$

$0x^3$

$\frac{x^5}{x} = x^4$

b) $(x^5 + 4x^4 + 3x^2 + 19x + 28) \div \boxed{(x + 4)}$

-4	1	4	0	3	19	28
		-4	0	0	-12	-28
	1	0	0	3	7	0

x^3 x^2

$x^4 + 3x + 7$

8.) (10 pts) For the function:

$x^4 + 8x^3 + 9x^2 - 38x - 40$

$p = 1$

± 1

$q = 40$

$1 \cdot 40$

$2 \cdot 20$

$4 \cdot 10$

$5 \cdot 8$

a) Find all potential zeros.

$\{ \underbrace{\pm 1 \;\; \pm 2} \;\; \pm 4 \;\; \pm 5 \;\; \pm 8 \;\; \pm 10 \;\; \pm 20 \;\; \pm 40$

b) Find the number of possible *positive* zeros.

1

$x^4 + 8x^3 + 9x^2 - 38x - 40$

1

c) Find the number of possible *negative* zeros.

4-1

$\boxed{3, 1}$

$x^4 + 8x^3 + 9x^2 - 38x - 40$

$x^4 - 8x^3 + 9x^2 + 38x - 40$

1 2 3

d) Attempt to find **3 zeros** using long division or synthetic division. Show all work.

$x^4 + 8x^3 + 9x^2 - 38x - 40$

✓ -1 | 1 8 9 -38 -40

-1 -7 -2 40

1 7 2 -40 0

$f(1) = -60$

1 | 1 8 9 -38 -40

1 9 18 -20

1 9 18 -20 -60

$f(-2) = 24$

-2 | 1 8 9 -38 -40

-2 -12 6 +64

1 6 -3 -32 24

$3-i$

9.) (10 pts) Find a polynomial of minimum degree with the following zeros:

x^3

-4, 3 - *i*, 3 + *i*

$x = 3-i$
$-3+i \quad -3+i$
$x-3-i=0$

$(x-3+i)(x-3-i)$

$x^2-3x-\cancel{ix}-3x+9+\cancel{3i}+\cancel{ix}-\cancel{3i}-i^2$

$x = 3+i$
$-3-i \quad -3-i$
$x-3-i=0$

$x^2-3x-3x+9-i^2$

$x^2-3x-3x+9-(\sqrt{-1})^2$ $\quad i=\sqrt{-1}$

$x^2-3x-3x+9-(-1)$

$x^2-3x-3x+9+1$

$x^2-6x+10 \; (x+4)$

$\boxed{(x+4)(x^2-6x+10)}$

$1+i,\ 1-i$

10.) (10 pts) Given a zero of the polynomial, determine all other zeros (real or complex) and write the polynomial as a product of linear factors.

$\{3i, -3i\}$

x^2+9

$x^4 + x^3 - 8x^2 + 4x - 48$, zero = $2i$

assume $-2i$

$(x-2i)(x+2i)$

$x^2+9=0$
$-9\ -9$

$x^2 + \cancel{2ix} - \cancel{2ix} - 4i^2$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

$x^2 - 4i^2$

$x^2 - 4(-1)$

x^2+4

$x = 2i \quad x = -2i$
$-2i\ -2i \quad +2i\ +2i$

x

$x - 2i = 0 \quad x + 2i = 0$

$$\begin{array}{r}
x^2 + x - 12 \\
x^2+0x+4 \overline{)\, x^4 + x^3 - 8x^2 + 4x - 48} \\
-x^4 - 0x^3 \mp 4x^2 \quad \downarrow \\
\hline
x^3 - 12x^2 + 4x \\
-x^3 \mp 0x^2 - 4x \\
\hline
-12x^2\ \ 0x\ -48 \\
+12x^2\ \ 0x\ +48 \\
\hline
0
\end{array}$$

$(x^2+4)(x^2+x-12)$

$$\boxed{(x^2+4)(x+4)(x-3)}$$

11.) (5 pts each, 10 pts total) Find the domain and asymptotes (vertical and horizontal) of each of the following rational functions.

a) $\frac{x^2-4}{3x^2-8x+4}$

$3x^2 = 3x \cdot x$

$3x^2 - 8x + 4 \neq 0$

$(3x-2)(x-2) \neq 0$

$\frac{\cancel{(x-2)}(x+2)}{(3x-2)\cancel{(x-2)}}$

$3x - 2 \neq 0$ $\quad$ $x - 2 \neq 0$

$+2 \quad +2$ $\quad$ $+2 \quad +2$

$\frac{3x}{3} \neq \frac{2}{3}$ $\quad$ $x \neq 2$

$x \neq \frac{2}{3}$

hole b/c it cancels

asymptote → $\frac{2}{3}$

hole → x

$\frac{x^2-4}{3x^2-8x+4}$

$\frac{1\cancel{x^2}}{3\cancel{x^2}} = \frac{1}{3}$

horizontal asymptote $= \frac{1}{3}$

b) $\frac{4x^2-3x+6}{8x^3-16x^2+8x}$