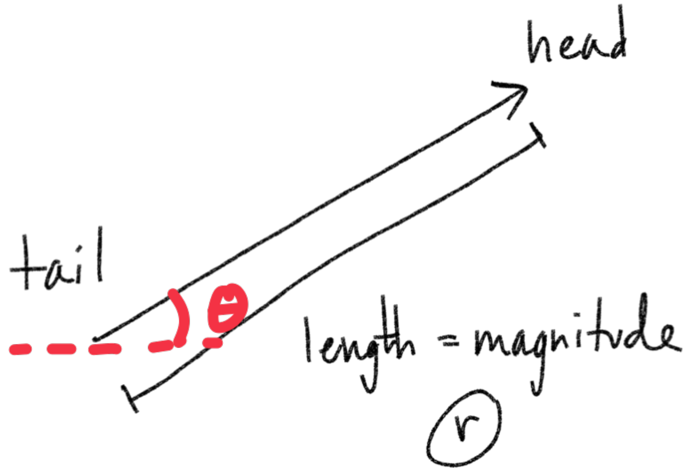
MTH-PT Trigonometry Session 25 4/29

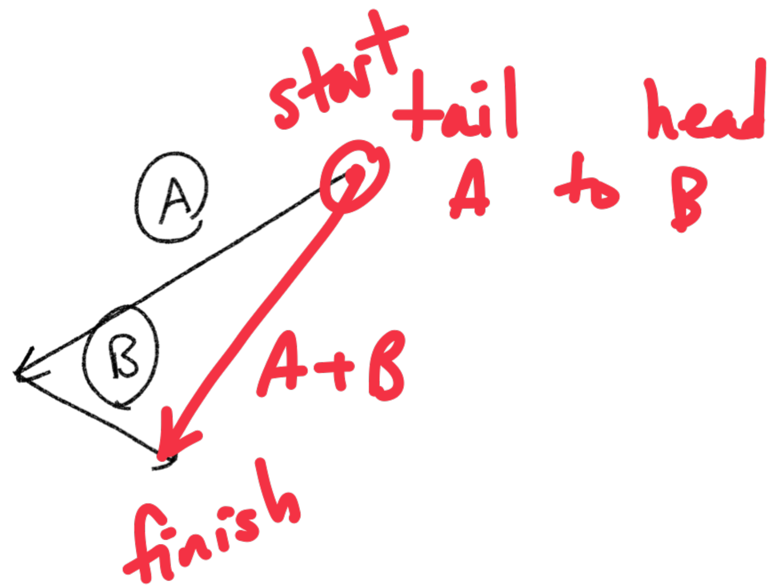
Vector → magnitude and a direction.



Polar Coordinates

(r, θ)

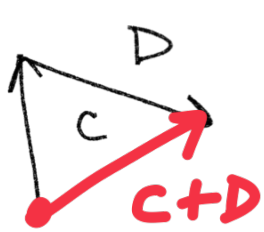
magnitude direction



A+B

Adding the tail of B to the head of A

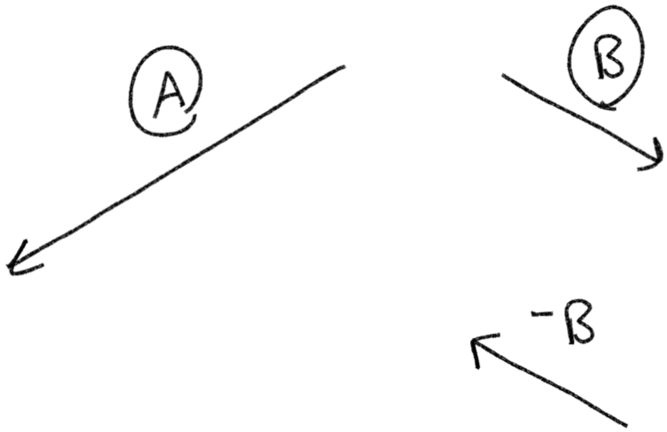
C+D

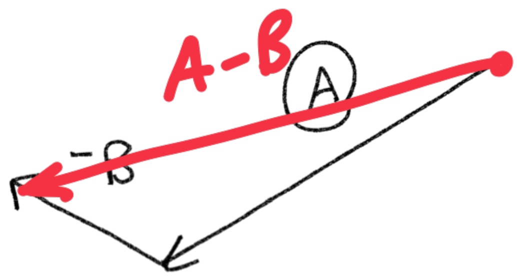


$A - B$

opposite

$A - B = \underbrace{A + (-B)}$

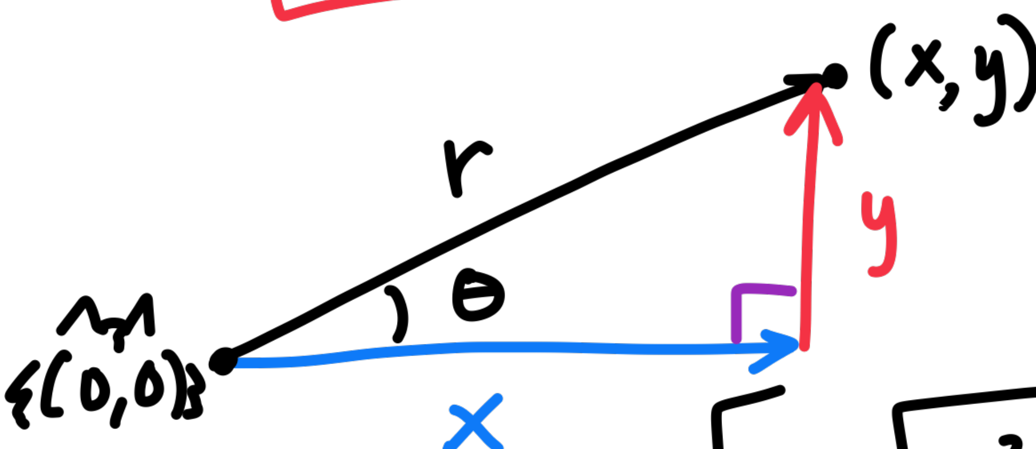




Vectors

$\boxed{x\text{- component}}$

$\boxed{y\text{- component}}$



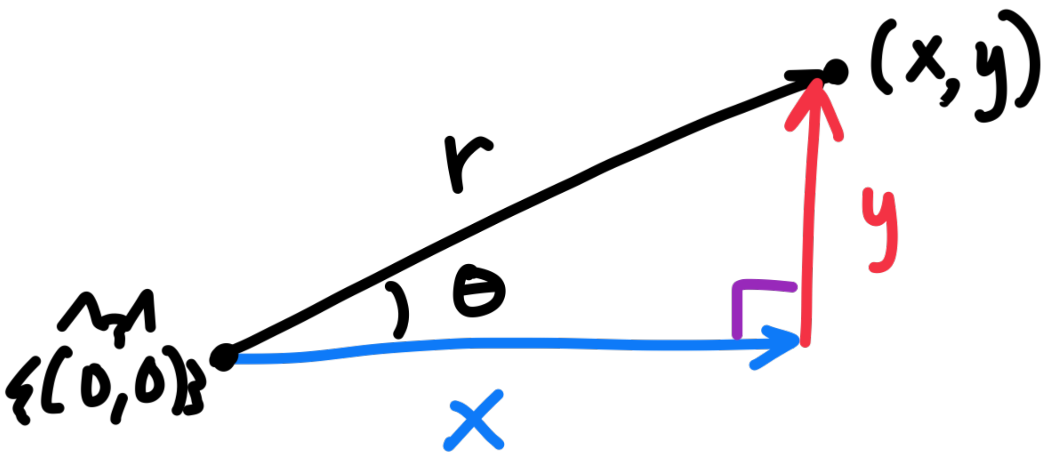
$$\left[\, r = \sqrt{x^2 + y^2} \,\right]$$

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

magnitude = r

$$\sqrt{(x - 0)^2 + (y - 0)^2}$$



$$\cos\theta = \frac{adj}{hyp} \qquad \sin\theta = \frac{opp}{hyp}$$

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

$$r(\cos\theta) = \left(\frac{x}{r}\right)r \qquad r(\sin\theta) = \left(\frac{y}{r}\right)r$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

For funsies...

In a unit circle,

radius (r) = 1

r = 1

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$(r)^2 = \left(\sqrt{x^2 + y^2}\right)^2$$

$$r^2 = x^2 + y^2$$

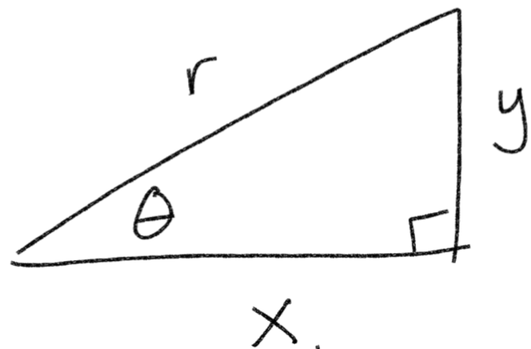
$$r^2 = (r\cos\theta)^2 + (r\sin\theta)^2$$

$$[1 = \cos^2\theta + \sin^2\theta]$$

vector (r, θ)

$$\boxed{r = \sqrt{x^2 + y^2}}$$

$$\boxed{\theta = \tan^{-1} \frac{y}{x}}$$

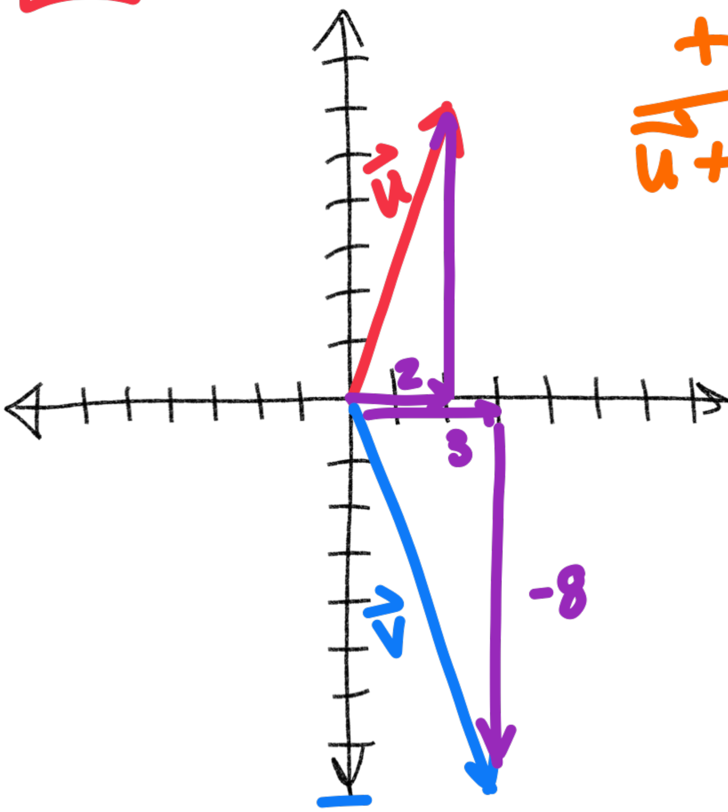


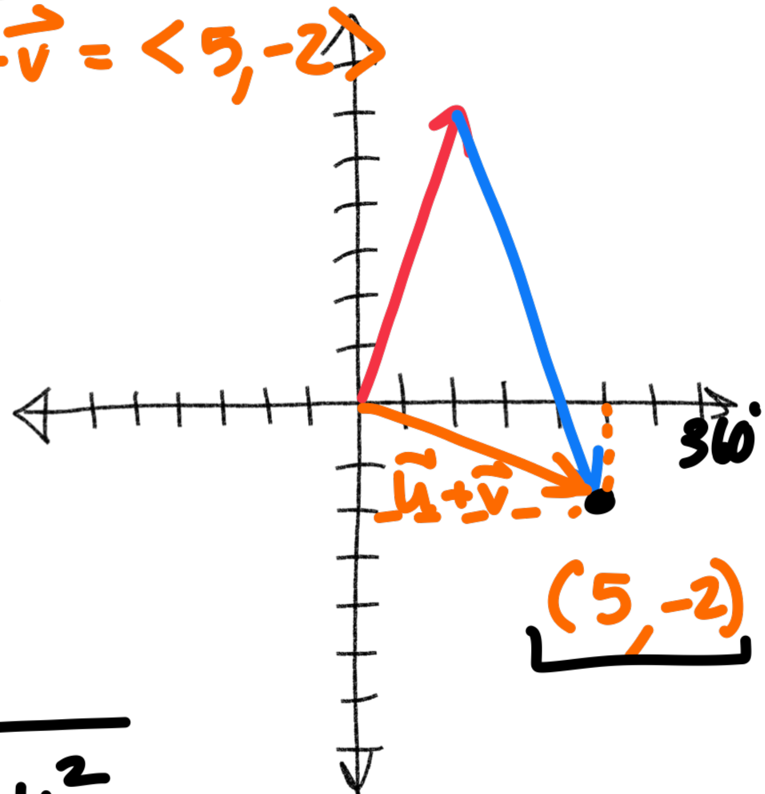
$$\tan^{-1} \tan\theta = \tan^{-1} \frac{y}{x}$$

$$\theta = \tan^{-1} \frac{y}{x}$$

$\boxed{\vec{u}} + \vec{v}$

$$\begin{aligned} \vec{u} &: \langle 2, 6 \rangle \\ + \vec{v} &: \langle 3, -8 \rangle \\ \hline \vec{u} + \vec{v} &= \langle 5, -2 \rangle \end{aligned}$$





$(5, -2)$

$\vec{u} + \vec{v}$

$$r = \sqrt{x^2 + y^2}$$

$$\sqrt{5^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-2}{5}\right) = -21.8$$

(r, θ)

$$\left[\begin{array}{l} (\sqrt{29}, 338.2°) \\ (\sqrt{29}, -21.8°) \end{array}\right]$$

$$\begin{array}{r} 360 \\ -\ 21.8 \\ \hline 338.2° \end{array}$$

$-\vec{u} + \vec{v}$

$\vec{u} = \langle 2, 6 \rangle$

$\vec{v} = \langle 3, -8 \rangle$

$-\vec{u} = \langle -2, -6 \rangle$

1.) Find components

2.) r $\quad r = \sqrt{x^2+y^2}$

3.) θ $\quad \theta = \tan^{-1}\left(\frac{y}{x}\right)$

1.) Find components

$\langle 1, -14 \rangle$

$$\begin{array}{r} \langle 3, -8 \rangle \\ + \langle -2, -6 \rangle \\ \hline \langle 1, -14 \rangle \end{array}$$

2.) $r = \sqrt{x^2+y^2}$

$\sqrt{(1)^2+(-14)^2} = \sqrt{1+196} = \sqrt{197} = 14.0$

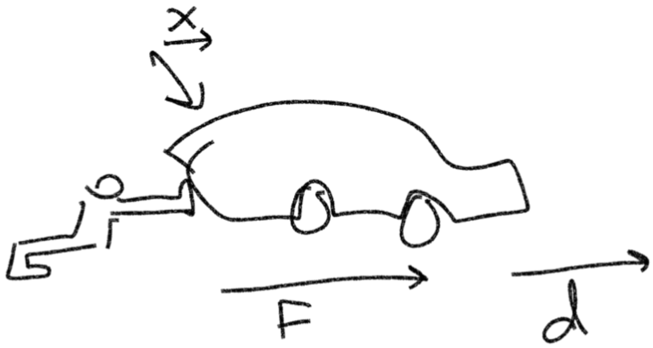
3.) $\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-14}{1}\right) = -85.9°$

$(14.0, -85.9°)$ or $(14.0, 274.1°)$

Work = Force • distance

Dot Product

product of the components



$\vec{u} \cdot \vec{V}$

$\vec{u} : \langle 2, 6 \rangle$

$\vec{V} : \langle 3, -8 \rangle$

$(u_x * V_x) + (u_y * V_y)$

$(2 * 3) + (6 * -8)$

$6 + (-48) = \boxed{-42}$

Angle between two vectors

$$\cos\theta = \frac{u \cdot V}{|u| * |v|}$$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u| * |v|}\right) = \cos^{-1}\left(\frac{-42}{(\sqrt{40})(\sqrt{73})}\right) = 141^\circ$$

$$|u| = \sqrt{x^2 + y^2} = \sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40}$$

$$|v| = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-8)^2} \quad \sqrt{9 + 64} = \sqrt{73}$$