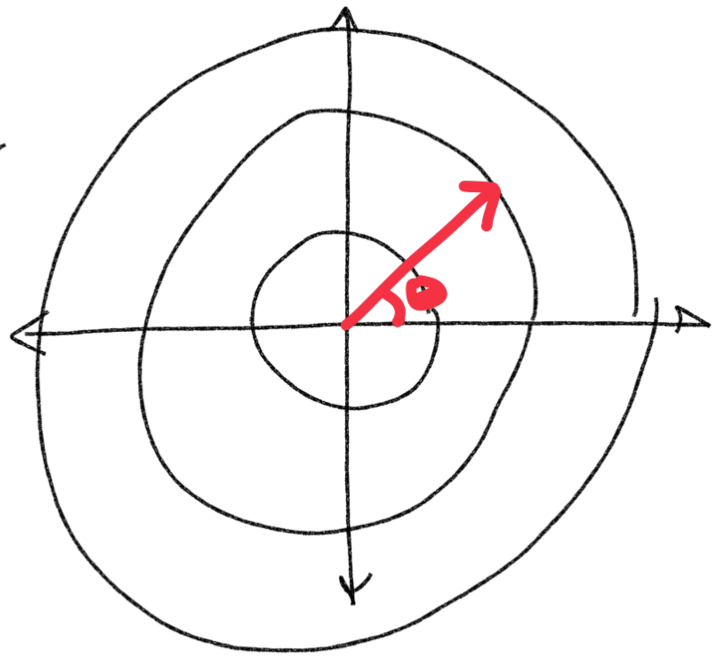
MTH-PT Trigonometry Session 26 5/2

Vector → Magnitude and direction (r, θ) polar

(x, y) rectangular

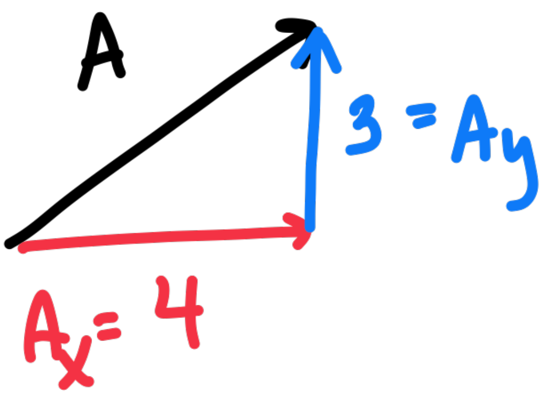


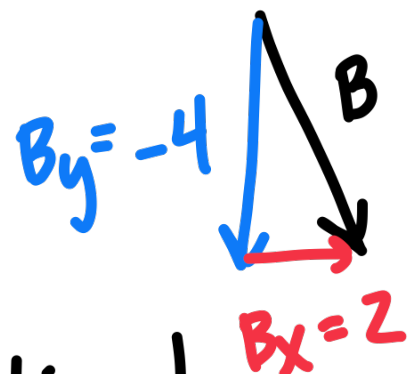
$(2, \theta)$

$(2, 60^\circ)$

$(2, \frac{\pi}{3})$

$60^\circ * \frac{\pi}{180^\circ}$





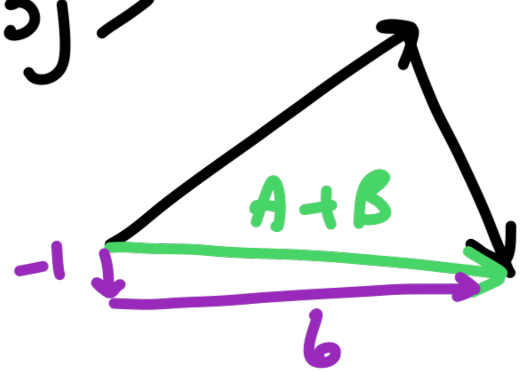
optional

$A: \langle 4, 3 \rangle$ or $\langle 4\hat{i} + 3\hat{j} \rangle$

$B: \langle 2, -4 \rangle$

$+$

$A+B: \langle 6, -1 \rangle$



$A+B \quad \langle 6, -1 \rangle$

$\text{Magnitude} = r = \sqrt{x^2+y^2}$

$\sqrt{(6)^2+(-1)^2}$

$\sqrt{36+1} = \sqrt{37}$

$\text{Direction: } \theta = \tan^{-1} \dfrac{y}{x}$

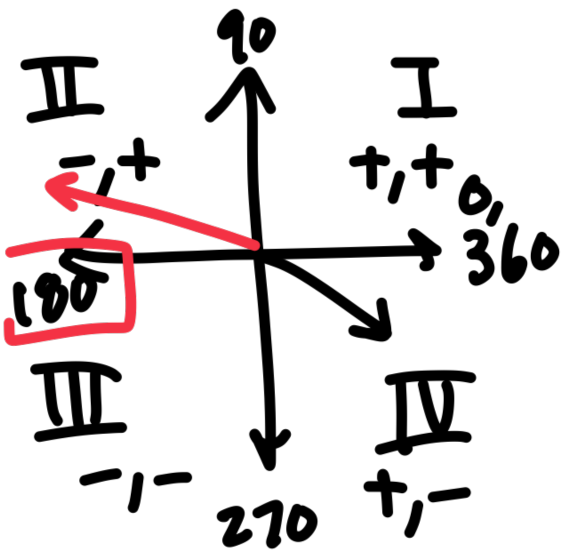
$\tan^{-1}\left(\dfrac{-1}{6}\right) = -9.5^\circ$

$(r, \theta) \quad (\sqrt{37}, -9.5^\circ)$

or

$(\sqrt{37}, 350.5^\circ)$

$$\begin{array}{r} 360 \\ -\ 9.5 \\ \hline 350.5^\circ \end{array}$$



If $(-6, 1)$

$\tan\left(\dfrac{1}{-6}\right) = -9.5$

$$\begin{array}{r} 180 \\ -\ 9.5 \\ \hline 170.5 \end{array}$$

$A : \langle 6, 4 \rangle$ Find $A+B \rightarrow (r, \theta)$

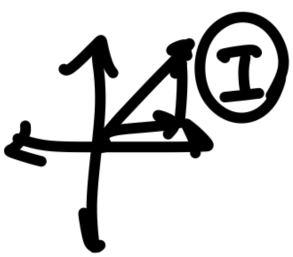
$B : \langle -2, 5 \rangle$

$A+B \quad \langle 4, 9 \rangle$

$$r = \sqrt{x^2 + y^2} = \sqrt{4^2 + 9^2}$$

$$\sqrt{16 + 81} = \sqrt{97}$$

$$\theta = \tan^{-1} \frac{y}{x} = \tan^{-1}\left(\frac{9}{4}\right) = 66^\circ$$



$(r, \theta) \quad (\sqrt{97}, 66^\circ)$

Work = F • d

$\vec{A} \cdot \vec{B}$

A: < -4, 8 >

B: < -2, -7 >

$(A_x * B_x) + (A_y * B_y)$

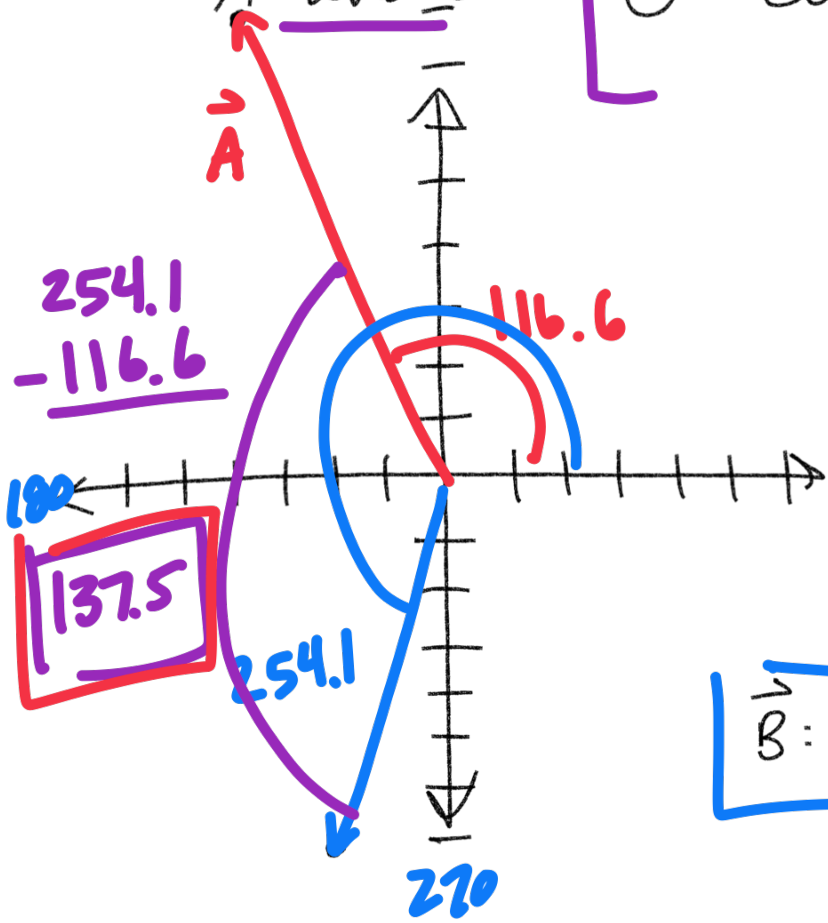
$(-4 * -2) + (8 * -7)$

$8 + (-56)$

$\boxed{-48}$

Angle between A and B

$$\left[\Theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|A|\,|B|} \right]$$



$\boxed{\vec{A} : (-4, 8)}$

$\Theta = \tan^{-1}\left(\frac{8}{-4}\right)$

$\tan^{-1}(-2)$

$-63.4°$

$180 - 63.4$

116.6

$\boxed{\vec{B} : (-2, -7)}$

$\Theta = \tan^{-1}\left(\frac{-7}{-2}\right)$

74.1

$180 + 74.1 = 254.1$

$A: (-4, 8)$

$B: (-2, -7)$

$\vec{A} \cdot \vec{B}$

$(-4 \cdot -2) + (8 \cdot -7)$

$8 + (-56) = -48$

Angle in Between A & B

$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$

$|A| = \sqrt{(A_x)^2 + (A_y)^2}$

$\sqrt{-4^2 + (8)^2}$

$\sqrt{16 + 64} = \sqrt{80}$

$\sqrt{80} = \sqrt{16} \cdot \sqrt{5}$

$= \boxed{4\sqrt{5}}$

$|B| = \sqrt{(B_x)^2 + (B_y)^2}$

$\sqrt{(-2)^2 + (-7)^2}$

$\sqrt{4 + 49}$

$\sqrt{53}$

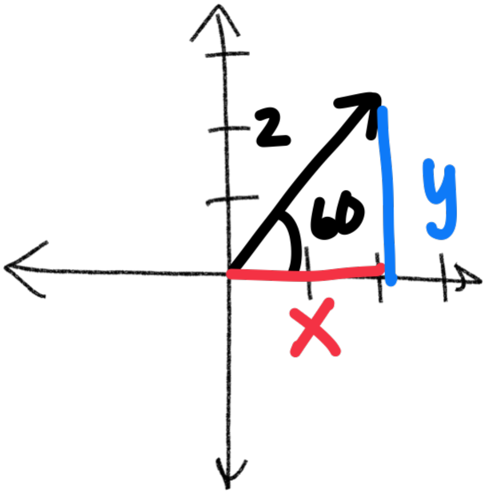
$\cos^{-1}\left(\frac{-48}{(4\sqrt{5})(\sqrt{53})}\right)$

$\boxed{137.5°}$

$(r, \theta) = (2, 60^\circ) \longrightarrow (\boxed{x}, \boxed{y})$

polar

rectangular



$2\left(\cos 60^\circ\right) = \left(\frac{x}{2}\right)2$

$x = 2 \cos 60$

$2\left(\frac{1}{2}\right) = 1$

$2\left(\sin 60^\circ\right) = \left(\frac{y}{2}\right)2$

$y = 2 \sin 60$

$2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$

(x, y)

$\boxed{(1, \sqrt{3})}$

$x = r \cos \theta$

$y = r \sin \theta$

$(r, \theta) \rightarrow [x, y]$

$[r\cos\theta, r\sin\theta]$

$(r, \theta) = (8, 120^\circ) \rightarrow [x, y]$

$\boxed{[-4, 4\sqrt{2}]}$

$x = 8 \cos 120 = -4$

$y = 8 \sin 120 = 4\sqrt{2}$

Rectangular $\longrightarrow$ Polar

$[x, y]$ $\quad$ (r, θ)

$[8, 6]$

$$\left(\sqrt{x^2+y^2},\ \tan^{-1}\left(\frac{y}{x}\right)\right)$$

(r, θ)

$\boxed{(10, 36.9^\circ)}$

$\sqrt{8^2+6^2}$ $\quad$ $\tan^{-1}\left(\frac{6}{8}\right)$

$\sqrt{64+36}$ $\quad$ 36.9

$\sqrt{100}$

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Convert $(2, 210^\circ) \rightarrow$ rectangular $[x, y]$

$x = r\cos\theta = 2\cos 210^\circ = -1.7$

$y = r\sin\theta = 2\sin 210^\circ = -1$

$\boxed{[-1.7, -1]}$

Convert $(-5, 12) \longrightarrow$ polar (r, θ)

$r = \sqrt{(-5)^2+(12)^2} = \sqrt{25+144}$ $\quad$ $\theta = \tan^{-1}\left(\frac{12}{-5}\right) = -67.4^\circ$

$\sqrt{169} = 13$ $\quad$ $\boxed{(13, 202.6)}$

$$\begin{array}{r} -67.4 \\ +270 \\ \hline 202.6 \end{array}$$