Name _____ Class _____ Date _____

Reteaching 1-1

Properties of Real Numbers

OBJECTIVE: Finding additive and multiplicative inverses

MATERIALS: None

The ***additive inverse*** of a number a is $-a$. The number $-a$ is also called the ***opposite*** of a. The sum of a number and its opposite, $a + (-a)$, is always 0.

The ***multiplicative inverse*** of a nonzero number a is $\frac{1}{a}$. The number $\frac{1}{a}$ is also called the reciprocal of a. The product of a nonzero number and its reciprocal, $a \cdot \frac{1}{a}$, is always 1. The number 0 does not have a multiplicative inverse.

Examples

Find the opposite and reciprocal of each number.

a. -7.4 **b.** $3\frac{1}{2}$

a. Opposite: $-(-7.4) = 7.4$

Reciprocal: $\frac{1}{-7.4} = \frac{10}{-74} = -\frac{10}{74} = -\frac{5}{37}$

b. Opposite: $-\left(3\frac{1}{2}\right) = -3\frac{1}{2}$

Reciprocal: $\frac{1}{3\frac{1}{2}} = \frac{1}{\frac{7}{2}} = \frac{2}{7}$

Exercises

Find the opposite and reciprocal of each number.

1. 3 **2.** -2 **3.** $-\frac{1}{6}$ **4.** $\frac{3}{5}$

5. -2.4 **6.** 0.6 **7.** $-5\frac{2}{3}$ **8.** $2\frac{1}{4}$

9. $\frac{\pi}{2}$ **10.** $-\frac{1}{\pi}$ **11.** -0.25 **12.** 1.3

13. $1\frac{2}{5}$ **14.** $-\sqrt{2}$ **15.** $\pi + 2$ **16.** $-\frac{9}{10}$

Name _____ Class _____ Date _____

Practice 1-1

Properties of Real Numbers

Simplify.

1. $-|4.2|$
2. $|12 - 16|$
3. $\left|-\frac{7}{6}\right|$
4. $|3| - |-2|$
5. $\left|\frac{2}{3}\right|$
6. $0.3|-6|$
7. $|14 - 8|$
8. $|-0.01|$

Replace each $ with the symbol <, >, or = to make the sentence true.

9. $-\sqrt{6}$ $ $\sqrt{10}$
10. $\frac{3}{2}$ $ 1.5
11. 0.06 $ 0.6
12. 4 $ $|-4|$
13. -0.4 $ 0
14. $-|-7|$ $ $|-7|$
15. 0.9 $ $\frac{2}{3}$
16. $\sqrt{2}$ $ $\sqrt{5}$

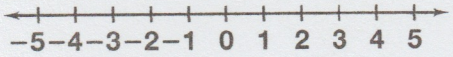
Name all the sets of numbers to which each number belongs.

17. -5
18. 0
19. $\sqrt{5}$
20. $2.\overline{7}$
21. 9
22. $\frac{10}{7}$
23. 1.2345267831 . . .
24. $-\frac{4}{2}$

Name the property of real numbers illustrated by each equation.

25. $\pi + 3 = 3 + \pi$
26. $\sqrt{2} + 0 = \sqrt{2}$
27. $(2 + x) + 3 = 2 + (x + 3)$
28. $\frac{5}{9} \cdot \frac{9}{5} = 1$
29. $16(3t + 4v) = 48t + 64v$
30. $\sqrt{2} \cdot 3 = 3 \cdot \sqrt{2}$
31. $0.01 \cdot 1 = 0.01$
32. $\frac{3}{2} \cdot \frac{2}{3} = 1$
33. $7 + (-7) = 0$
34. $2(xy) = (2x)y$

Graph the number on the following number line. Estimate if necessary.



35. $-\sqrt{2}$
36. $\frac{3}{2}$
37. 0.5
38. -1

Find the opposite and the reciprocal of each number.

39. $-2\frac{1}{2}$
40. 3
41. $\frac{5}{9}$
42. -4

Which set of numbers best describes the values of each variable?

43. the number of stops N a commuter train makes on a certain day

44. the high H and low L for a certain stock during a period of n weeks

45. the average time per lap t it takes a race car to complete n laps

Name _____ Class _____ Date _____

Reteaching 1-2

Algebraic Expressions

OBJECTIVE: Simplifying and evaluating algebraic expressions

MATERIALS: None

To simplify an algebraic expression, combine like terms using the basic properties of real numbers. Like terms have the same variables raised to the same powers.

To evaluate an algebraic expression, replace the variables in the expression with numbers and follow the order of operations.

Example

Simplify the algebraic expression $3(4x + 5y) - 2(3x - 7y)$. Then evaluate the simplified expression for $x = 3$ and $y = -2$.

Simplify the algebraic expression using the basic properties of real numbers.

$$\begin{aligned}
3(4x + 5y) - 2(3x - 7y) &= 3(4x + 5y) + (-2)(3x + (-7)y) && \leftarrow \textbf{definition of subtraction} \\
&= 12x + 15y + (-6)x + 14y && \leftarrow \textbf{Distributive Property} \\
&= 12x + (-6)x + 15y + 14y && \leftarrow \textbf{Commutative Property of Addition} \\
&= (12 + (-6))x + (15 + 14)y && \leftarrow \textbf{Distributive Property} \\
&= 6x + 29y
\end{aligned}$$

Now replace x with 3 and y with -2 in the simplified expression.

$6(3) + 29(-2) = 18 - 58 = -40$

Exercises

Simplify the algebraic expression. Then evaluate the simplified expression for the given values of the variable.

1. $(4x + 1) + 2x; x = 3$
2. $7(t + 3) - 11; t = 4$
3. $3y + 4z + 6y - 9z; y = 2, z = 1$
4. $2(u + v) - (u - v); u = 8, v = -3$
5. $5a^2 + 5a + a + 1; a = -2$
6. $6p^2 - (3p^2 + 2q^2); p = 1, q = 5$
7. $\frac{3}{4}(m + n) - \frac{1}{4}(m - n); m = 6, n = 2$
8. $\frac{r}{2} + \frac{s}{3} - \frac{r}{4} + \frac{1}{5}; r = -1, s = 0$

Name _____ Class _____ Date _____

Practice 1-2

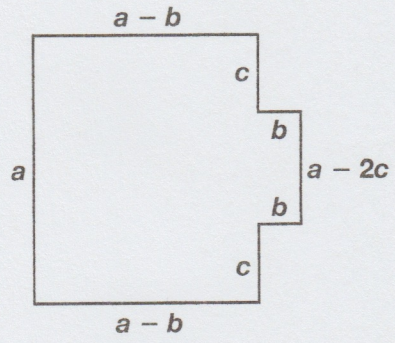
Algebraic Expressions

Simplify by combining like terms.

1. $6x + x$
2. $11t + 3t - 5$
3. $-6a - 5a + b - 1$
4. $5i + 7j - 3i$
5. $16xy - 4xy$
6. $5x - 3x^2 + 16x^2$
7. $3(m - 2) + m$
8. $\frac{3(a - b)}{9} + \frac{4}{9}b$
9. $t + \frac{t^2}{2} + t^2 + t$
10. $4a - 5(a + 1)$
11. $2(m - n^2) - 6(n^2 + 3m)$
12. $x(x - y) + y(y - x)$
13. The expression $6s^2$ represents the surface area of a cube with edges of length s. Find the surface area of a cube with each edge length.
    a. 3 inches
    b. 1.5 meters
14. The expression $4.95 + 0.07x$ models a household's monthly long-distance charges, where x represents the number of minutes of long-distance calls during the month. Find the monthly charges for 73 minutes.

Evaluate each expression for the given value of the variable.

15. $5y^2 + y + 1; y = 4$
16. $a + 6 + 3a; a = 5$
17. $-t^2 - (3t + 2); t = 5$
18. $i^2 - 5(i^3 - i^2); i = 7$
19. $k + 2 - 4k - 1; k = -3$
20. $6a - 3a^2 - 2a^3; a = 1$
21. $-m(2m + m^2); m = -4$
22. $3 - 2n - 5 + n^2; n = -3$
23. $12b - 3 + b^2; b = 9$
24. $a^2 + b^2; a = 3, b = 4$
25. $c(3 - a) - c^2; a = 4, c = -1$
26. $-a^2 + 3(d - 2a); a = 2, d = -3$
27. Write an expression for the perimeter of the figure as the sum of the lengths of its sides. Then simplify your answer.



Name _____ Class _____ Date _____

Reteaching 1-3

Solving Equations

OBJECTIVE: Solving an equation for one of its variables

MATERIALS: None

To solve an equation for one of its variables, rewrite the equation as an equivalent equation with the specified variable on one side of the equation by itself and an expression not containing that variable on the other side.

Example

Solve the equation $\frac{ax - b}{2} = x + 2b$ for x.

Use the properties of equality and the properties of real numbers to rewrite the equation as a sequence of equivalent equations.

$$\frac{ax - b}{2} = x + 2b$$

$2\left(\frac{ax - b}{2}\right) = 2(x + 2b)$ ← **Multiply each side by 2.**

$ax - b = 2(x + 2b)$ ← **Simplify.**

$ax - b = 2x + 4b$ ← **Distributive Property**

$ax - 2x = 4b + b$ ← **Add and subtract to get terms with x on one side and terms without x on the other side.**

$ax - 2x = 5b$ ← **Simplify.**

$x(a - 2) = 5b$ ← **Distributive Property**

$x = \frac{5b}{a - 2}$ ← **Divide each side by $a - 2$.**

The final form of the equation has x on the left side by itself and an expression not containing x on the right side.

Exercises

Solve each equation for the indicated variable.

1. $3m - n = 2m + n$, for m
2. $2(u + 3v) = w - 5u$, for u
3. $ax + b = cx + d$, for x
4. $k(y + 3z) = 4(y - 5)$, for y
5. $\frac{1}{2}r + 3s = 1$, for r
6. $\frac{2}{3}f + \frac{5}{12}g = 1 - fg$, for f
7. $\frac{x + k}{j} = \frac{3}{4}$, for x
8. $\frac{a - 3y}{b} + 4 = a + y$, for y

Name _____ Class _____ Date _____

Practice 1-3

Solving Equations

Solve each formula for the indicated variable.

1. $V = \frac{\pi}{3} r^2 h$, for h
2. $S = L(1 - r)$, for r
3. $S = \ell w + wh + \ell h$, for w

Solve for x. State any restrictions on the variables.

4. $\frac{4}{9}(x + 3) = g$
5. $a(x + c) = b(x - c)$
6. $\frac{x + 3}{t} = t^2$

7. Two brothers are saving money to buy tickets to a concert. Their combined savings is \$55. One brother has \$15 more than the other. How much has each saved?

8. The sides of a triangle are in the ratio 5 : 12 : 13. What is the length of each side of the triangle if the perimeter of the triangle is 15 in.?

9. Find three consecutive numbers whose sum is 126.

Solve each equation.

10. $\frac{1}{2}(x - 3) + \left(\frac{3}{2} - x\right) = 5x$
11. $5w + 8 - 12w = 16 - 15w$
12. $7y + 5 = 6y + 11$
13. $1.2(x + 5) = 1.6(2x + 5)$
14. $t - 3\left(t + \frac{4}{3}\right) = 2t + 3$
15. $0.5(c + 2.8) - c = 0.6c + 0.3$
16. $3(x + 1) = 2(x + 11)$
17. $\frac{u}{5} + \frac{u}{10} - \frac{u}{6} = 1$

18. Mike and Adam left a bus terminal at the same time and traveled in opposite directions. Mike's bus was in heavy traffic and had to travel 20 mi/h slower than Adam's bus. After 3 hours, their buses were 270 miles apart. How fast was each bus going?

19. Two trains left a station at the same time. One traveled north at a certain speed and the other traveled south at twice the speed. After 4 hours, the trains were 600 miles apart. How fast was each train traveling?

20. Find four consecutive odd integers whose sum is 336.

21. The length of a rectangle is 5 cm greater than its width. The perimeter is 58 cm. Find the dimensions of the rectangle.

Name _____ Class _____ Date _____

Reteaching 1-4

Solving Inequalities

OBJECTIVE: Solving and graphing inequalities **MATERIALS:** None

To solve an inequality, use the techniques used to solve an equation with one difference: when multiplying or dividing each side by a negative number, reverse the inequality.

Examples

Solve each inequality. Graph the solutions.

a. $2x - 5 \geq 13$ **b.** $4 + 3(1 - 2x) > 37$

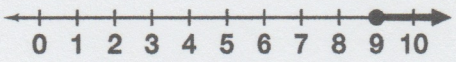
Use the properties of real numbers and the properties of inequalities to rewrite each inequality in equivalent forms.

a. When dividing each side by a positive number, do not reverse the inequality.

$2x - 5 \geq 13$

$2x \geq 18$ ← **Add 5 to each side.**

$x \geq 9$ ← **Divide each side by 2.**



b. When dividing each side by a negative number, reverse the inequality.

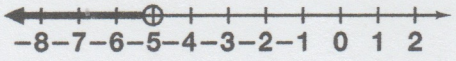
$4 + 3(1 - 2x) > 37$

$4 + 3 - 6x > 37$ ← **Distributive Property**

$7 - 6x > 37$ ← **Simplify.**

$-6x > 30$ ← **Subtract 7 from each side.**

$x < -5$ ← **Divide each side by –6 and reverse the inequality.**



Exercises

Solve each inequality. Graph the solutions.

1. $3(y - 5) \leq 6$ **2.** $-4t > 2$ **3.** $3 - 4m < 11$ **4.** $7d \leq 2(d + 5)$

5. $-2(3 - h) + 2h \geq 0$ **6.** $3k - (1 - 2k) > 1$ **7.** $5p + 12 \leq 9p - 20$ **8.** $3 - 2r < 7 - r$

Name _____ Class _____ Date _____

Practice 1-4

Solving Inequalities

Solve each inequality. Graph the solutions.

1. $16 - 4t \leq 36$
2. $2(m + 3) + 1 > 23$
3. $7 + 13(x + 1) \leq 3x$
4. $-6a < 21$
5. $\frac{2}{3}(4x + 5) > \frac{9}{4}x$
6. $2[5x - (3x - 4)] < 3(2x + 3)$
7. $8(x - 5) \geq 56$
8. $6 - x \leq 7x + 3$
9. $10 - x \geq -2(3 + x)$

Solve each compound inequality. Graph the solutions.

10. $-9 \leq 4x + 3 \leq 11$
11. $16x \leq 32$ or $-5x < -40$
12. $9x < 54$ and $-4x < 12$
13. $6(x + 2) \geq 24$ or $5x + 10 \leq 15$
14. $14 > 3x - 1 \geq -10$
15. $4 < 1 - 3x < 7$
16. $2(x - 1) < -4$ or $2(x - 1) > 4$
17. $3x - 5 \geq -8$ and $3x - 5 \leq 1$

Solve each problem by writing an inequality.

18. A salesperson earns \$350 per week plus 10% of her weekly sales. Find the sales necessary for the salesperson to earn at least \$800 in one week.
19. The length of a rectangular yard is 50 ft, and its perimeter is less than 170 ft. Describe the width of the yard.
20. Xul is two years older than his sister Maria. The sum of their ages is greater than 32. Describe Maria's age.
21. A research team estimates that 30% of their questionnaires will not be returned. How many questionnaires should they mail out in order to be reasonably certain that at least 750 will be returned?

Solve each problem by writing a compound inequality.

22. Watermelons cost \$.39 per pound at a local market. Kent's watermelon cost between \$4.00 and \$5.00. What are the possible weights of his watermelon?
23. How much must a carpenter cut off a 48-inch board if the length must be 40 ± 0.25 inches?
24. A concrete slab requires between 10 and 12 yd^3 of concrete. If 2.5 yd^3 of concrete can be poured each hour, how long will it take to pour the slab?

Name _____ Class _____ Date _____

Reteaching 1-5

Absolute Value Equations and Inequalities

OBJECTIVE: Solving absolute value equations **MATERIALS:** None

For every positive real number a, both a and $-a$ satisfy the equation $|x| = a$.

To solve an absolute value equation, first rewrite the equation as an equivalent equation with the absolute value expression on the left side by itself. Then rewrite this equation as a compound equality using the rule that if $|x| = a$ then $x = a$ or $x = -a$.

Example

Solve the equation $2|x - 3| + 1 = 6x + 7$. Check for extraneous solutions.

Use the properties of equality to rewrite the equation as an equivalent equation with the absolute value expression on one side by itself. Then write that equation as a compound equality and solve each resulting equation.

$2|x - 3| + 1 = 6x + 7$

$2|x - 3| = 6x + 6$ ← **Subtract 1 from each side.**

$|x - 3| = 3x + 3$ ← **Divide each side by 2.**

$x - 3 = 3x + 3$ or $x - 3 = -(3x + 3)$ ← **Rewrite as a compound equality.**

$-2x = 6$ or $x - 3 = -3x - 3$ ← **Solve each equation.**

$x = -3$ or $4x = 0$

$x = -3$ or $x = 0$

To check for extraneous solutions, substitute each value for x in the original absolute value equation. Any value that does not satisfy the original equation must be discarded.

Check $2|-3 - 3| + 1 \stackrel{?}{=} 6(-3) + 7$

$2|-6| + 1 \stackrel{?}{=} -18 + 7$

$2(6) + 1 \stackrel{?}{=} -11$

$13 \neq -11$

$2|0 - 3| + 1 \stackrel{?}{=} 6(0) + 7$

$2|-3| + 1 \stackrel{?}{=} 0 + 7$

$2(3) + 1 \stackrel{?}{=} 7$

$7 = 7$

The only solution is 0; -3 is an extraneous solution.

Exercises

Solve each equation. Check for extraneous solutions.

1. $|2x + 7| = 5$ **2.** $|x - 3| = -1$ **3.** $|x + 7| = 2x + 8$ **4.** $|x - 0.5| + 0.3 = 1$

5. $3|2x + 5| = 15$ **6.** $|5x - 1| + 7 = 3x$ **7.** $2|x + 1| + x = 1$ **8.** $|x + 1| = 2x$

Name _____ Class _____ Date _____

Practice 1-5

Absolute Value Equations and Inequalities

Write each specification as an absolute value inequality.

1. $6.3 \leq h \leq 10.3$
2. $-2.5 \leq a \leq 2.5$
3. $22 \leq x \leq 33$

Solve each inequality. Graph the solutions.

4. $|x + 5| > 12$
5. $|k - 3| \leq 19$
6. $|x + 2| \geq 0$
7. $2|t - 5| < 14$
8. $|3x - 2| + 7 \geq 11$
9. $5|2b + 1| - 3 \leq 7$
10. $|2 - 3w| \geq 4$
11. $-3|7m - 8| < 5$
12. $|2u| > 6$

Solve each equation. Check for extraneous solutions.

13. $|4x| = 28$
14. $|3x + 6| = -12$
15. $|z - 1| = 7z - 13$
16. $|s + 12| = 15$
17. $|-3x| = 63$
18. $2|5x + 3| = 16$
19. $|6x + 7| = 5x + 2$
20. $|7r - 4| = 24$
21. $|3c| + 2 = 11$
22. $5|x + 1| + 6 = 21$
23. $|3x + 5| - 2x = 3x + 4$
24. $-|d + 2| = 7$

Write an absolute value inequality and a compound inequality for each length x with the given tolerance.

25. a length of 4.2 cm with a tolerance of 0.01 cm
26. a length of 3.5 m with a tolerance of 0.2 cm
27. a length of 10 ft with a tolerance of 1 in.
28. Write an absolute value inequality and a compound inequality for the temperature T that was recorded to be as low as 65°F and as high as 87°F on a certain day.
29. The weight of a 40-lb bag of fertilizer varies as much as 4 oz from the stated weight. Write an absolute value inequality and a compound inequality for the weight w of a bag of fertilizer.
30. The duration of a telephone call to a software company's help desk is at least 2.5 minutes and at most 25 minutes. Write an absolute value inequality and a compound inequality for the duration d of a telephone call.

Name _____ Class _____ Date _____

Reteaching 1-6

Probability

OBJECTIVE: Finding theoretical probability **MATERIALS:** None

The possible results of an experiment are **outcomes.** If you want to find the theoretical probability of a particular event, or a **favorable outcome,** you use this formula:

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{number of possible outcomes}}$$

Example

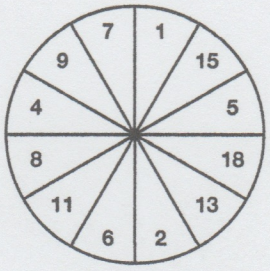
Find the theoretical probability of rolling a number cube and having an outcome of either 2 or 4.

$$P(2 \text{ or } 4) = \frac{(\text{number of times 2 or 4 are outcomes})}{(\text{total possible numbers on cube})} = \frac{2}{6}$$

$$= \frac{1}{3}$$

Exercises

Use the spinner at the right to determine the theoretical probability for each event.



1. P(the number is even)
2. P(5)
3. P(the number is prime)
4. P(the number is less than 6)
5. P(an odd number)
6. P(a number divisible by 2)
7. P(a multiple of 3)
8. P(an 11 or 15)
9. P(a composite number)
10. P(the number represents your age)
11. P(a perfect square)
12. P(the number represents your grade)
13. P(not a 5 or 7)

Name _____ Class _____ Date _____

Practice 1-6

Probability

1. You select a number at random from the sample space {1, 2, 3, 4, 5}. Find each theoretical probability.
   a. P(the number is 2)
   b. P(the number is even)
   c. P(the number is prime)
   d. P(the number is less than 5)

2. In a class of 19 students, 10 study Spanish, 7 study French, and 2 study both French and Spanish. One student is picked at random. Find each probability.
   a. P(studying Spanish but not French)
   b. P(studying neither Spanish nor French)
   c. P(studying both Spanish and French)
   d. P(studying French)

3. In a telephone survey of 150 households, 75 respondents answered "Yes" to a particular question, 50 answered "No," and 25 were "Not sure." Find each experimental probability.
   a. P(answer was "Yes")
   b. P(answer was "No")
   c. P(answer was "Not sure")
   d. P(answer was not "Not sure")

4. A wallet contains four bills with denominations of \$1, \$5, \$10, and \$20. You choose two of the four bills from the wallet at random and add the dollar amounts.
   a. What is the sample space? How many outcomes are there?
   b. What is the probability of getting \$15?
   c. What is the probability of getting \$50?
   d. What is the probability of getting at least \$25?

5. A basketball player has attempted 24 shots and made 13. Find the experimental probability that the player will make the next shot that she attempts.

6. A baseball player attempted to steal a base 70 times and was successful 47 times. Find the experimental probability that the player will be successful on his next attempt to steal a base.

For Exercises 7–8, define a simulation by telling how you represent correct answers, incorrect answers, and the quiz. Use your simulation to find each experimental probability.

7. If you guess the answers at random, what is the probability of getting at least three correct answers on a four-question true-false quiz?

8. A five-question multiple-choice quiz has four choices for each answer. If you guess the answers at random, what is the probability of getting at least four correct answers?

9. A circular pool of radius 12 ft is enclosed within a rectangular yard measuring 50 ft by 100 ft. If a ball from an adjacent golf course lands at a random point within the yard, what is the probability that the ball lands in the pool?

10. Five people each flip a coin. What is the theoretical probability that all five will get heads?