**Graphing Systems of Equations** 

**OBJECTIVE:** Solving a system by graphing

**MATERIALS:** One blue and one yellow highlighter

As you solve a system of equations, remember the following ideas.

- Lines that have the same slopes but different *y*-intercepts are parallel and will never intersect. These systems are *inconsistent*.
- Lines that have both the same slope and the same y-intercept are the same line and will intersect at every point. These systems are dependent.
- Lines that have different slopes will intersect, and the system will have one solution. These systems are *independent*.

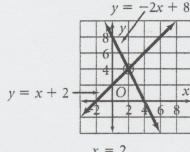
### Example

Solve the system of equations by graphing.  $\begin{cases} 2x + y = 8 \\ y - x = 2 \end{cases}$ 

$$y = -2x + 8$$

 $\leftarrow$  Write both equations in y = mx + b form.

$$y = x + 2$$



Graph the line y = -2x + 8 with a blue highlighter. Graph the line y = x + 2 with a yellow highlighter. The point of intersection will be green. Circle it.

x = 2y = 4

**←** Determine the x- and y-coordinates of the point of intersection.

Check by substituting the solution into both equations.

The solution is the ordered pair (2, 4).

$$2(2) + 4 \stackrel{?}{=} 8$$

$$8 = 8 \checkmark$$

$$4 - 2 \stackrel{?}{=} 2$$

$$2 = 2$$

### Exercises

Use colored highlighters to solve each system of equations by graphing.

1. 
$$\begin{cases} 3x + y = 6 \\ y = 3 \end{cases}$$

$$2. \begin{cases} -2x + y + 3 = 0 \\ x - 1 = y \end{cases}$$

3. 
$$\begin{cases} x + y = 3 \\ y = 3x - 1 \end{cases}$$

**4.** 
$$\begin{cases} y = 1 - x \\ 2x + y = 4 \end{cases}$$

5. 
$$\begin{cases} -x + 2y = 2 \\ 3x + 2y = -6 \end{cases}$$

**6.** 
$$\begin{cases} -x + y = -2 \\ -2x + 3y = -3 \end{cases}$$

**Graphing Systems of Equations** 

Classify each system without graphing.

1. 
$$\begin{cases} x + y = 3 \\ y = 2x - 3 \end{cases}$$

$$2. \begin{cases} 2x + y = 3 \\ y = -2x - 3 \end{cases}$$

3. 
$$\begin{cases} x + 3y = 9 \\ -2x - 6y = -18 \end{cases}$$

**4.** 
$$\begin{cases} x + y = 4 \\ y = 2x + 1 \end{cases}$$

2. 
$$\begin{cases} 2x + y = 3 \\ y = -2x - 1 \end{cases}$$
5. 
$$\begin{cases} x + 3y = 9 \\ 9y + 3x = 27 \end{cases}$$

**6.** 
$$\begin{cases} x + 2y = 5 \\ 2x + 3y = 9 \end{cases}$$

7. 
$$\begin{cases} 3x + 2y = 7 \\ 3x - 15 = -6y \end{cases}$$
10. 
$$\begin{cases} x + 2y = 13 \\ 2y = 7 - x \end{cases}$$

8. 
$$\begin{cases} x + y = 6 \\ 3x + 3y = 3 \end{cases}$$
11. 
$$\begin{cases} y = 12 - 5x \\ x - 4y = -6 \end{cases}$$

**9.** 
$$\begin{cases} x + y = 11 \\ y = x - 5 \end{cases}$$

**10.** 
$$\begin{cases} x + 2y = 13 \\ 2y = 7 - x \end{cases}$$

**11.** 
$$\begin{cases} y = 12 - 5x \\ x - 4y = -6 \end{cases}$$

9. 
$$\begin{cases} x + y = 11 \\ y = x - 5 \end{cases}$$
12. 
$$\begin{cases} 25x - 10y = 0 \\ 2y = 5x \end{cases}$$

- 13. The spreadsheet below shows the monthly income and expenses for a new business.
  - a. Find a linear model for monthly income and a linear model for monthly expenses.
  - b. Use the models to estimate the month in which income will equal expenses.

	A Month	B	C Expenses
1	May	\$1500	\$21,400
2	June	\$3500	\$18,800
3	July	\$5500	\$16,200
4	August	\$7500	\$13,600

Solve each system by graphing. Check your answers.

**14.** 
$$\begin{cases} y = x - 2 \\ x + y = 10 \end{cases}$$

**15.** 
$$\begin{cases} y = 7 - x \\ x + 3y = 11 \end{cases}$$

**16.** 
$$\begin{cases} x - 2y = 10 \\ y = x - 11 \end{cases}$$

**17.** 
$$\begin{cases} 5x + y = 11 \\ x - y = 1 \end{cases}$$

**18.** 
$$\begin{cases} x + y = -1 \\ x - y = 3 \end{cases}$$

$$19. \begin{cases} x - y = -1 \\ 2x + 2y = 10 \end{cases}$$

**20.** 
$$\begin{cases} 4x + 3y = -16 \\ -x + y = 4 \end{cases}$$

**21.** 
$$\begin{cases} y = -3x \\ x + y = 2 \end{cases}$$

19. 
$$\begin{cases} x - y = -1 \\ 2x + 2y = 10 \end{cases}$$
22. 
$$\begin{cases} y = \frac{2}{3}x - 5 \\ y = -\frac{2}{3}x - 3 \end{cases}$$

23. 
$$\begin{cases} y = \frac{1}{2}x + 3 \\ y = -\frac{1}{4}x - 3 \end{cases}$$

**24.** 
$$\begin{cases} 2x - 4y = -4 \\ 3x - y = 4 \end{cases}$$

**25.** 
$$\begin{cases} x + y = 6 \\ x - y = 4 \end{cases}$$

# Solving systems - Substitution Method © 2012 Kuta Software LLC. All rights reserved.

Solve each system by substitution. SHOW ALL YOUR WORK.

1) 
$$7x + 4y = 10$$
  
 $y = x - 14$ 

2) 
$$y = x - 5$$
  
 $-3x - y = -3$ 

3) 
$$-x + 7y = -17$$
  
 $y = 6x - 20$ 

4) 
$$y = 4x - 16$$
  
 $-x - 6y = -4$ 

5) 
$$y = 2x + 6$$
  
 $-3x - 5y = 22$ 

6) 
$$y = -3x - 16$$
  
 $5x - 2y = -23$ 

7) 
$$y = 5x - 17$$
  
 $4x - 2y = 22$ 

8) 
$$y = x + 15$$
  
 $7x + 6y = -14$ 

9) 
$$y = -2x + 1$$
  
 $-4x - 5y = 1$ 

10) 
$$5x - 3y = -9$$
  
 $y = 8x + 3$ 

11) 
$$y = 6x + 15$$
  
 $2x + 7y = 17$ 

12) 
$$-6x - y = 19$$
  
 $y = -7x - 21$ 

13) 
$$y = x + 4$$
  
 $-8x + 2y = 2$ 

14) -4x + 2y = -10y = -3x + 5

15) 
$$-4x + 5y = 13$$
  
 $7x + y = -13$ 

16) x - 6y = -204x - 2y = 8

17) 
$$-3x + y = 7$$
  
 $-4x + 4y = -12$ 

18) -6x + 5y = -82x + y = 24

19) 
$$x - 4y = -2$$
  
 $4x - 4y = -20$ 

20) -6x - 2y = 16x + 7y = 4

21) 
$$-4x + y = -10$$
  
  $3x + 7y = 23$ 

22) x - 3y = -165x + 7y = -14

23) 
$$-8x - 7y = -8$$
  
 $x - 2y = 1$ 

24) 6x - 6y = 03x + y = -12

25) 
$$-4x - 4y = -8$$
  
  $x + 8y = 16$ 

26) 
$$6x + 3y = -3$$
  
 $x - 3y = -4$ 

#### **Preview of the Lesson:**

1.

2.

3.

**II. WHAT IS A SYSTEM OF EQUATIONS?** 

**III. WHAT IS THE SOLUTION TO A SYSTEM OF EQUATIONS?** 

IV. ADDING/SUBTRACTING EQUATIONS

a. 
$$2x + 3y = 12$$
  
 $2x + 4y = 16$ 

b. 
$$-7x + 2y = 15$$
  
 $6x + 4y = 24$ 

**Eliminating Variables...looking for opposites!** 

c. 
$$5x + 2y = 10$$
  
 $-5x + 4y = 30$ 

d. 
$$7x - 4y = -28$$
  
 $3x + 4y = 12$ 

#### **MULTIPLYING AN EQUATION BY A CONSTANT TO ELIMINATE A VARIABLE**

e. 
$$6x + 3y = 24$$
  
 $2x + 3y = -12$ 

f. 
$$x + 2y = 3$$
  
 $x - 8y = -16$ 

g. 
$$-5x + 4y = 20$$
  
 $15x + 9y = -45$ 

h. 
$$6x - 4y = -24$$
  
 $9x - 2y = 18$ 

1. Add/subtract by \_\_\_\_\_terms.

2. Remember to include \_\_\_\_\_ sides of the equation when multiplying.

V. STEPS FOR SOLVING SYSTEMS OF EQUATIONS ALGEBRAICALLY BY ELIMINATION:

**VI. GUIDED EXAMPLES:** 

1. 
$$3x + 2y = 15$$
  
 $-3x + 5y = 6$ 

You try

2. 
$$-2x - 6y = -6$$
  
2x + y = 9

Check your answers:

## 3.3 – Solving Systems of Equations by Elimination

Write your questions and thoughts here!

3. 
$$-6x - 6y = -6$$
  
 $-4x - 6y = -6$ 

$$4. -4x + 6y = -16$$
$$-4x + 3y = -16$$

5. 
$$15x + 4y = -22$$
  
 $-5x + 8y = 26$ 

6. 
$$4x - y = -5$$
  
 $8x + 3y = -5$ 

7. 
$$-3x - 3y = -30$$
  
 $-2x + 7y = -2$ 

$$8. \quad -3x + 8y = 28$$
$$-4x + 6y = 28$$

### VII - Special Cases:



9. 
$$-12x - 6y = -18$$
  
 $6x + 3y = 0$ 

10. 
$$-18x + 12y = 6$$
  
 $9x - 6y = -3$ 

### **Additional Notes**

### **Practice Exercises**

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#### Solve each system by elimination.

1) 
$$4x - 6y = -12$$
  
 $-4x - 2y = 28$ 

$$-4x - 2y = 28$$

3) 
$$-5x - 7y = -12$$
  
 $7x - 7y = 0$ 

5) 
$$-6x - 2y = -4$$
  
 $-12x - 5y = -13$ 

7) 
$$3x + 6y = 6$$
  
 $10x + 10y = -20$ 

9) 
$$5x + 15y = 20$$
  
 $-27y = 9x$ 

2) 
$$-6x + 8y = 2$$
  
 $6x - 5y = -8$ 

4) 
$$7x - 8y = -8$$
  
 $7x - 5y = 16$ 

6) 
$$-4x + 4y = 8$$
  
 $9x - 8y = -13$ 

8) 
$$10x + 5y = 25$$
  
 $-7x - 3y = -18$ 

10) 
$$-10x - 4y = 4$$
  
 $-5x - 2y = 2$ 

**OBJECTIVE:** Solving systems of inequalities

MATERIALS: Graph paper and a graphing calculator (optional)

### Example

Solve the system  $\begin{cases} 2x - y > 1 \\ x + y \ge 3 \end{cases}$  by graphing.

#### Step 1

Solve each inequality for y.

$$2x - y > 1$$

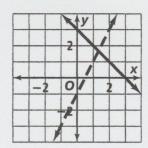
$$-y > -2x + 1$$

$$y < 2x - 1$$
and
$$x + y \ge 3$$

 $y \ge -x + 3$ 

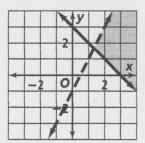
#### Step 2

Graph the boundary lines. Use a solid line for  $\geq$  or  $\leq$  inequalities. Use a dotted line for > and < inequalities.



#### Step 3

Then shade on the appropriate side of each boundary line.



### Exercises

Solve each system of inequalities by graphing.

$$1. \begin{cases} y \le x \\ y \ge 3x - 1 \end{cases}$$

$$2. \begin{cases} 2x + y > 3 \\ x - y < 2 \end{cases}$$

4. 
$$\begin{cases} x + 3y \le 9 \\ 2x - y > 1 \end{cases}$$

5. 
$$\begin{cases} y < -\frac{1}{3}x - 1 \\ y \ge 3x + 1 \end{cases}$$
8. 
$$\begin{cases} 2x + y < 3 \\ 3x - y < 2 \end{cases}$$

7. 
$$\begin{cases} y \le 3 \\ y > 4x - 3 \end{cases}$$

8. 
$$\begin{cases} 2x + y < 3 \\ 2x + y < 3 \end{cases}$$

$$3. \begin{cases} x > 1 \\ y < x + 1 \end{cases}$$

**6.** 
$$\begin{cases} 4x + y \le 1 \\ x + 2y \le -1 \end{cases}$$

$$9. \begin{cases} y \ge -2 \\ 3x + y \le 2 \end{cases}$$

**Solving Systems of Inequalities** 

Solve each system of inequalities by graphing.

$$1. \begin{cases} y > x + 2 \\ y \le -x + 1 \end{cases}$$

$$2. \begin{cases} y \le x + 3 \\ y \ge x + 2 \end{cases}$$

3. 
$$\begin{cases} x + y < 5 \\ y < 3x - 2 \end{cases}$$

4. 
$$\begin{cases} x - 2y < 3 \\ 2x + y > 8 \end{cases}$$

5. 
$$\begin{cases} -3x + y < 3 \\ x + y > -1 \end{cases}$$

**6.** 
$$\begin{cases} x + 2y > 4 \\ 2x - y > 6 \end{cases}$$

7. 
$$\begin{cases} 2x \ge y + 3 \\ x < 3 - 2y \end{cases}$$

**8.** 
$$\begin{cases} 3 < 2x - y \\ x - 3y \le 4 \end{cases}$$

$$9. \begin{cases} y \ge 2 \\ y \ge |x| \end{cases}$$

10. 
$$\begin{cases} y < x - 3 \\ y \ge |x - 4| \end{cases}$$

11. 
$$\begin{cases} -2x + y > 1 \\ y > |x| \end{cases}$$

12. 
$$\begin{cases} y < -3 \\ y < -|x| \end{cases}$$

- 13. Suppose you are buying two kinds of notebooks for school. A spiral notebook costs \$2, and a three-ring notebook costs \$5. You must have at least six notebooks. The cost of the notebooks can be no more than \$20.
  - a. Write a system of inequalities to model the situation.
  - b. Graph and solve the system.
- **14.** A camp counselor needs no more than 30 campers to sign up for two mountain hikes. The counselor needs at least 10 campers on the low trail and at least 5 campers on the high trail.
  - a. Write a system of inequalities to model the situation.
  - b. Graph and solve the system.

Solve each system of inequalities by graphing.

**15.** 
$$\begin{cases} 2x + y > 2 \\ x - y \ge 3 \end{cases}$$

$$16. \begin{cases} y \le 3x \\ y \ge -2x + 2 \end{cases}$$

17. 
$$\begin{cases} y < 5x - 1 \\ y \ge 7 - 3x \end{cases}$$

**18.** 
$$\begin{cases} y \ge -2x + 2 \\ y \le 3x \end{cases}$$

**19.** 
$$\begin{cases} x + y > 2 \\ 2x - y < 1 \end{cases}$$

**20.** 
$$\begin{cases} y > 3x + 2 \\ y \le -2x + 1 \end{cases}$$

**21.** 
$$\begin{cases} y \ge -2 \\ y \le -|x+3| \end{cases}$$

**22.** 
$$\begin{cases} y < x + 3 \\ y > |x - 1| \end{cases}$$

**23.** 
$$\begin{cases} y > x \\ y < |x+2| \end{cases}$$

**Linear Programming** 

**OBJECTIVE:** Solving linear programming

MATERIALS: Graph paper

problems

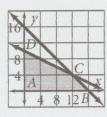
### Example

Use linear programming. Find the values of x and y that maximize and minimize the objective function P = 10x + 15y.

Restrictions 
$$\begin{cases} x + y \le 16 \\ 3x + 6y \le 60 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

#### Step 1

Graph the restrictions.



#### Step 2

Find coordinates of each vertex of the region.

#### VERTEX

A(0,0)

B(16,0)

C(12,4)

D(0, 10)

Evaluate P at each vertex.

$$P = 10x + 15y$$

$$P = 10(0) + 15(0) = 0$$

$$P = 10(16) + 15(0) = 160$$

$$P = 10(12) + 15(4) = 180$$

$$P = 10(0) + 15(10) = 150$$

The maximum value of the objective function is 180. It occurs when x = 12and y = 4.

The minimum value of the objective function is 0. It occurs when x = 0 and y=0.

#### **Exercises**

Use linear programming. Find the values of x and y that maximize and minimize each objective function.

1. 
$$\begin{cases} 5y + 4x \le 35 \\ 5y + x \ge 20 \\ y \le 6 \\ x \ge 1 \end{cases}$$

$$P = 8x + 2y$$

$$2. \begin{cases} x + y \ge \\ x \ge y \\ x \le 4 \\ y \ge 0 \end{cases}$$

$$P = x + 3y$$

3. 
$$\begin{cases} 3x + 4y \ge 12 \\ 5x + 6y \le 30 \\ 1 \le x \le 3 \end{cases}$$

$$P = x - 2y$$

**Linear Programming** 

Graph each system of constraints. Name all vertices. Then find the values of x and y that maximize or minimize the objective function.

$$1. \begin{cases} x + 2y \le 6 \\ x \ge 2 \\ y \ge 1 \end{cases}$$

$$2. \begin{cases} x + y \le 5 \\ x + 2y \le 8 \\ x \ge 0, y \ge 0 \end{cases}$$

3. 
$$\begin{cases} x + y \le 6 \\ 2x + y \le 10 \\ x \ge 0, y \ge 0 \end{cases}$$

Minimum for C = 3x + 4y

Maximum for 
$$P = x + 3y$$

Maximum for P = 4x + y

4. 
$$\begin{cases} 3x + 2y \le 6 \\ 2x + 3y \le 6 \\ x \ge 0, y \ge 0 \end{cases}$$

5. 
$$\begin{cases} 4x + 2y \le 4 \\ 2x + 4y \le 4 \\ x \ge 0, y \ge 0 \end{cases}$$

6. 
$$\begin{cases} x + y \le 5 \\ 4x + y \le 8 \\ x \ge 0, y \ge 0 \end{cases}$$

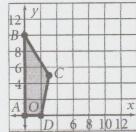
Maximum for P = 4x + y

Maximum for 
$$\bar{P} = 3x + y$$

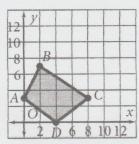
Minimum for C = x + 3y

Find the values of x and y that maximize or minimize the objective function for each graph. Then find the maximum or minimum value.

7.



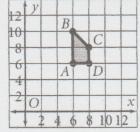
8.



Maximize for P = 2x + 3y

Minimize for C = x + 2y





Maximize for P = 3x + y

- 10. You are going to make and sell bread. A loaf of Irish soda bread is made with 2 c flour and  $\frac{1}{4}$  c sugar. Kugelhopf cake is made with 4 c flour and 1 c sugar. You will make a profit of \$1.50 on each loaf of Irish soda bread and a profit of \$4 on each Kugelhopf cake. You have 16 c flour and 3 c sugar.
  - **a.** How many of each kind of bread should you make to maximize the profit?
  - **b.** What is the maximum profit?
- 11. Suppose you make and sell skin lotion. A quart of regular skin lotion contains 2 c oil and 1 c cocoa butter. A quart of extra-rich skin lotion contains 1 c oil and 2 c cocoa butter. You will make a profit of \$10/qt on regular lotion and a profit of \$8/qt on extra-rich lotion. You have 24 c oil and 18 c cocoa butter.
  - **a.** How many quarts of each type of lotion should you make to maximize your profit?
  - **b.** What is the maximum profit?

**Graphs in Three Dimensions** 

**OBJECTIVE:** Graphing points in three

MATERIALS: Cardboard box

dimensions

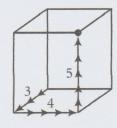
Use the inside corner of a cardboard box to model a three-dimensional coordinate system in which x, y, and z are all greater than 0.

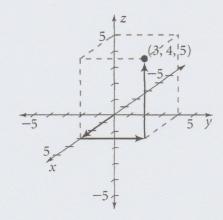
- First label the bottom (or horizontal) edges of the box as the x-axis and the y-axis.
- Then the z-axis will be the vertical edge that passes through the origin or corner where the three edges meet.



Example

Graph (3, 4, 5) in three dimensions. First, locate the point using the cardboard box. Start at the back left corner of the box. Move forward three units. Move right four units. Move up five units. Then graph the point.





**Exercises** 

Locate and graph each point in three dimensions by following the steps in the example.

- **1.** (1, 2, 0)
- **2.** (3, 2, 3)
- **3.** (4, 0, 3)
- 4. (5, 1, 2)

- **5.** (2, 5, 1)
- **6.** (0, 0, 4)
- **7.** (3, -1, -1)
- 8. (-3, -3, -3)

- 9. (5, 0, 1)
- **10.** (0, 1, 3)
- **11.** (1, 2, -5)
- **12.** (5, 3, -4)

- **13.** (2, 4, 6)
- **14.** (-2, -2, -2)
- **15.** (-1,-2,5)
- **16.** (1, 2, 8)

- 17. (-4, -5, 1)
- **18.** (0, 0, 2)
- **19.** (1, -4, -5)
- **20.** (-6, 2, -4)

**Graphs in Three Dimensions** 

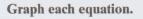
Describe the location of each point in coordinate space.

8. 
$$(0, 4, -1)$$

Graph each point in coordinate space.

**12.** 
$$(-1, -4, -2)$$

Write the coordinates of each point in the diagram.



**25.** 
$$x + 2y + 3z = 3$$

**26.** 
$$3x - 2y + z = 6$$

**27.** 
$$-6x - 3y + 2z = 6$$

**28.** 
$$2x - 3y + 3z = 6$$

**29.** 
$$8x - 2y - 2z = 8$$

**30.** 
$$-6x - 12y - 12z = 12$$

**31.** 
$$9x - 3y + z = 9$$

**32.** 
$$7x - 1y + 7z = 7$$

**33.** 
$$4x + 3y + 6z = 12$$

**34.** 
$$x - y + 2z = 6$$

#### Graph each equation and find the equation of each trace.

**35.** 
$$x + y + z = 3$$

**36.** 
$$x + 2y + 3z = 6$$

**37.** 
$$x + 3y + 2z = 6$$

**38.** 
$$2x + 3y + z = 6$$

**39.** 
$$-4x + 2y - 4z = 8$$

**40.** 
$$4x - 2y + 6z = 12$$

**41.** 
$$6x - 3y + z = 6$$

**42.** 
$$7x - 3y + 7z = 21$$

**43.** 
$$4x - 3y + 6z = -12$$

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**OBJECTIVE:** Using elimination to solve systems with three variables

MATERIALS: Pink, yellow, and green highlighting markers

### Example

Solve the system using elimination.

$$\begin{cases} x & + & y & + & z & = & 6 \\ 2x & - & y & + & 3z & = & 9 \\ -x & + & 2y & + & 2z & = & 9 \end{cases}$$

$$\begin{array}{rcl}
3y & + & 3z & = & 15 \\
-3y & + & (-7z) & = & -27 \\
\hline
& -4z & = & -12 \\
z & = & 3
\end{array}$$

$$3y + 3(3) = 15$$
  
 $3y = 6$   
 $y = 2$   
 $x + 2 + 3 = 6$ 

The solution is (1, 2, 3).

$$\leftarrow$$
 Multiply the green equation by 2 to eliminate  $x$  and add the two equations. Circle the resulting sum.

$$\leftarrow$$
 Substitute the value of z into either of the circled equations.

$$\leftarrow$$
 Solve for y.

Substitute the values of y and z into any of the original equations. Solve for x.

### Exercises

Solve each system using elimination.

1. 
$$\begin{cases} 2x - y + 2z = 10 \\ 4x + 2y - 5z = 10 \\ x - 3y + 5z = 8 \end{cases}$$
 2. 
$$\begin{cases} x - y + z = 6 \\ 2x + 3y + 2z = 2 \\ 3x + 5y + 4z = 4 \end{cases}$$

4. 
$$\begin{cases} 3x + y + z = 2 \\ 4x - 2y + 3z = -4 \\ 2x + 2y + 2z = 8 \end{cases}$$
5. 
$$\begin{cases} 5x + 2y + z = 5 \\ 3x - 3y - 3z = 9 \\ x + 2y + 4z = 6 \end{cases}$$

2. 
$$\begin{cases} x - y + z = 6 \\ 2x + 3y + 2z = 2 \\ 3x + 5y + 4z = 4 \end{cases}$$

5. 
$$\begin{cases} 5x + 2y + z = 5 \\ 3x - 3y - 3z = 9 \\ x + 2y + 4z = 6 \end{cases}$$

3. 
$$\begin{cases} 6x - 4y + 5z = 31 \\ 5x + 2y + 2z = 13 \\ x + y + z = 2 \end{cases}$$

6. 
$$\begin{cases} x + y + z = -1 \\ 4x + 3y + 2z = -10 \\ 2x - 4y - 2z = -6 \end{cases}$$

Systems with Three Variables

Solve each system.

1. 
$$\begin{cases} x + y + z = -1 \\ 2x - y + 2z = -5 \\ -x + 2y - z = 4 \end{cases}$$

2. 
$$\begin{cases} x + y + z = 3 \\ 2x - y + 2z = 6 \\ 3x + 2y - z = 13 \end{cases}$$

3. 
$$\begin{cases} 2x + y = 9 \\ x - 2z = -3 \\ 2y + 3z = 15 \end{cases}$$

4. 
$$\begin{cases} x - y + 2z = 10 \\ -x + y - 2z = 5 \\ 3x - 3y + 6z = -2 \end{cases}$$

5. 
$$\begin{cases} 2x - y + z = -4 \\ 3x + y - 2z = 0 \\ 3x - y = -4 \end{cases}$$

6. 
$$\begin{cases} 2x - y - z = 4 \\ -x + 2y + z = 1 \\ 3x + y + z = 16 \end{cases}$$

7. 
$$\begin{cases} x + 5y + 5z = -10 \\ x + y + z = 2 \\ x + 2y + 3z = -3 \end{cases}$$

8. 
$$\begin{cases} x - y - z = 0 \\ x - 2y - 2z = 3 \\ -2x + 2y - z = 3 \end{cases}$$

9. 
$$\begin{cases} 3x + y + z = 6 \\ 3x - 2y + 2z = 14 \\ 3x + 3y - 3z = -6 \end{cases}$$

10. 
$$\begin{cases} x + y + z = -2 \\ 2x + 2y - 3z = 11 \\ 3x - y + z = 4 \end{cases}$$

11. 
$$\begin{cases} x - 5y + z = 3 \\ x + 2y - 2z = -12 \\ 2x + 2z = 6 \end{cases}$$

**12.** 
$$\begin{cases} 2x & + 3z = 2 \\ 3x + 6y & = 6 \\ x & -2z = 8 \end{cases}$$

**13.** 
$$\begin{cases} x + y - z = 0 \\ 3x - y + z = 4 \\ 5x + z = 7 \end{cases}$$

**14.** 
$$\begin{cases} x - 2y &= 1 \\ x + 3y + z &= 0 \\ 2x & -2z &= 18 \end{cases}$$

**15.** 
$$\begin{cases} x + y + 4z = 5 \\ -2x + 2z = 3 \\ 3x + y - 2z = 0 \end{cases}$$

$$\begin{cases}
3x + 2y + 2z = 4 \\
-6x + 4y - 2z = -9 \\
9x - 2y + 2z = 10
\end{cases}$$

17. 
$$\begin{cases} 2x - 3y + z = -3 \\ x - 5y + 7z = -11 \\ -10x + 4y - 6z = 28 \end{cases}$$

**18.** 
$$\begin{cases} x + y + z = -8 \\ x - y - z = 6 \\ 2x - 3y + 2z = -1 \end{cases}$$

19. 
$$\begin{cases} 14x - 3y + 5z = -15 \\ 3x + 2y - 6z = 10 \\ 7x - y + 4z = -5 \end{cases}$$

20. 
$$\begin{cases} 5x - 3y + 2z = 39 \\ 4x + 4y - 3z = 34 \\ 3x - 2y + 6z = 14 \end{cases}$$

21. 
$$\begin{cases} x + y + z = 6 \\ 2x - y + 2z = 6 \\ -x + y + 3z = 10 \end{cases}$$

**22.** 
$$\begin{cases} 2x + y - z = 3 \\ 3x - y + 3z = 3 \\ -x - 3y + 2z = 3 \end{cases}$$

23. 
$$\begin{cases} 2x - 3y + z = 4 \\ -2x + 3y - z = -4 \\ 6x - 9y + 3z = 12 \end{cases}$$

**24.** 
$$\begin{cases} x + y - z = 1 \\ x + 2z = 3 \\ 2x + 2y = 4 \end{cases}$$

Write and solve a system of equations for each problem.

- **25.** The sum of three numbers is -2. The sum of three times the first number, twice the second number, and the third number is 9. The difference between the second number and half the third number is 10. Find the numbers.
- **26.** Monica has \$1, \$5, and \$10 bills in her wallet that are worth \$96. If she had one more \$1 bill, she would have just as many \$1 bills as \$5 and \$10 bills combined. She has 23 bills total. How many of each denomination does she have?