Name _____ Class _____ Date _____

Reteaching 5-1

Modeling Data with Quadratic Functions

OBJECTIVE: Recognizing and using a quadratic function

MATERIALS: Graphing calculator

A quadratic function can be written in standard form:

$f(x) = ax^2 + bx + c$, where $a \neq 0$.

ax^2	bx	c
quadratic term	linear term	constant term

Example

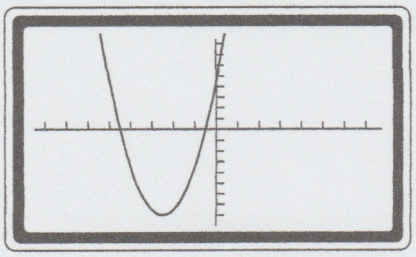
Rewrite the function in standard form. Indicate whether the function is quadratic. Graph the function to check your answer.

$f(x) = (1 + x)(9 + x)$ ← **Multiply and simplify to put in standard form.**

$f(x) = 9 + x + 9x + x^2$ ← **Use the FOIL Method to apply the Distributive Property.**

$f(x) = x^2 + 10x + 9$ ← **Combine like terms and simplify.**

Since it has a quadratic term, this is a quadratic function.



← **Use a graphing calculator to check your answer. This is a quadratic function, since its graph is a parabola.**

Exercises

Rewrite each function in standard form. Indicate whether the function is quadratic. Then graph to check.

1. $f(x) = (-5x - 4)(-5x - 4)$
2. $y = 3(x - 1) + 3$
3. $y = x^2 + 24 - 11x - x^2$
4. $g(x) = (x - 7)(x + 7)$
5. $f(x) = (3 - x)(x + 3)$
6. $g(x) = x^2$
7. $f(x) = 3x(x + 1) - x$
8. $f(x) = (x + 4)(x - 4)$
9. $f(x) = 4x^2 + 5x$
10. $y = 2(x + 2)^2 - 2x^2$

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Practice 5-1

Modeling Data with Quadratic Functions

Find a quadratic model for each set of values.

1. $(-1, 1), (1, 1), (3, 9)$

2. $(-4, 8), (-1, 5), (1, 13)$

3. $(-1, 10), (2, 4), (3, -6)$

4.

x	–1	0	2
$f(x)$	1	–1	7

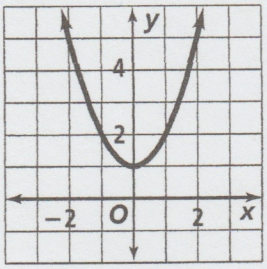
5.

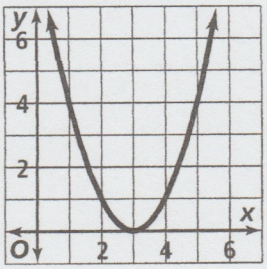
x	–4	0	1
$f(x)$	1	9	16

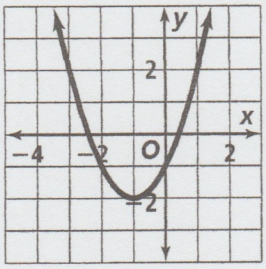
6.

x	–1	2	3
$f(x)$	12	3	4

Identify the vertex and the axis of symmetry of each parabola.

7. 

8. 

9. 

Determine whether each function is linear or quadratic. Identify the quadratic, linear, and constant terms.

10. $y = (x - 2)(x + 4)$

11. $y = 3x(x + 5)$

12. $y = 5x(x - 5) - 5x^2$

13. $f(x) = 7(x - 2) + 5(3x)$

14. $f(x) = 3x^2 - (4x - 8)$

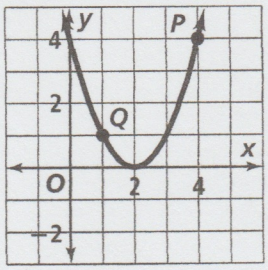
15. $y = 3x(x - 1) - (3x + 7)$

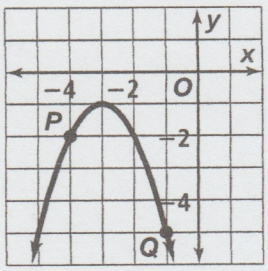
16. $y = 3x^2 - 12$

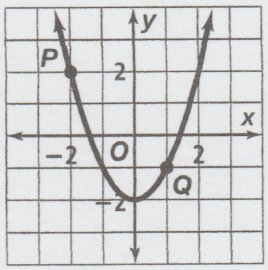
17. $f(x) = (2x - 3)(x + 2)$

18. $y = 3x - 5$

For each parabola, identify points corresponding to P and Q.

19. 

20. 

21. 

22. A toy rocket is shot upward from ground level. The table shows the height of the rocket at different times.

Time (seconds)	0	1	2	3	4
Height (feet)	0	256	480	672	832

a. Find a quadratic model for this data.

b. Use the model to estimate the height of the rocket after 1.5 seconds.

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Reteaching 5-2

Properties of Parabolas

OBJECTIVE: Graphing a parabola using the vertex and axis of symmetry

MATERIALS: Graph paper

- The graph of a quadratic function, $y = ax^2 + bx + c$, where $a \neq 0$, is a parabola.
- The axis of symmetry is the line $x = -\frac{b}{2a}$.
- The x-coordinate of the vertex is $-\frac{b}{2a}$. The y-coordinate of the vertex is $y = f\left(-\frac{b}{2a}\right)$, or the y-value when $x = -\frac{b}{2a}$.
- The y-intercept is $(0, c)$.

Example

Graph $y = 2x^2 - 8x + 5$.

$x = -\frac{b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$ ← **Find the equation of the axis of symmetry.**

x-coordinate of vertex: 2 ← $-\frac{b}{2a}$

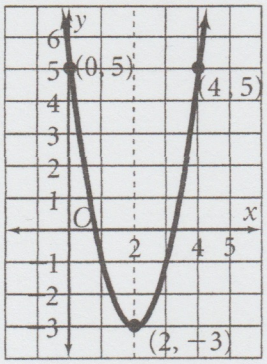
$f\left(-\frac{b}{2a}\right) = f(2) = 2(2)^2 - 8(2) + 5$ ← **Find the y-value when $x = 2$.**

$= 8 - 16 + 5$

$= -3$

y-coordinate of vertex: -3 ← **The vertex is at $(2, -3)$.**

y-intercept: $(0, 5)$ ← **The y-intercept is at $(0, c) = (0, 5)$.**



← **Since a is positive, the graph opens upward, and the vertex is at the bottom of the graph. Plot the vertex and draw the axis of symmetry. Plot (0, 5) and its corresponding point on the other side of the axis of symmetry.**

Exercises

Graph each parabola. Label the vertex and the axis of symmetry.

1. $y = x^2 - 4x + 7$
2. $y = x^2 + 8x + 11$
3. $y = -3x^2 + 6x - 9$
4. $y = -x^2 - 8x - 15$
5. $y = 2x^2 - 8x + 1$
6. $y = -2x^2 - 12x - 7$

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Practice 5-2

Properties of Parabolas

Graph each function. If $a > 0$, find the minimum value. If $a < 0$, find the maximum value.

1. $y = -x^2 + 2x + 3$
2. $y = 2x^2 + 4x - 3$
3. $y = -3x^2 + 4x$
4. $y = x^2 - 4x + 1$
5. $y = -x^2 - x + 1$
6. $y = 5x^2 - 3$
7. $y = \frac{1}{2}x^2 - x - 4$
8. $y = 5x^2 - 10x - 4$
9. $y = 3x^2 - 12x - 4$

Graph each function.

10. $y = x^2 + 3$
11. $y = x^2 - 4$
12. $y = x^2 + 2x + 1$
13. $y = 2x^2 - 1$
14. $y = -3x^2 + 12x - 8$
15. $y = \frac{1}{3}x^2 + 2x - 1$

16. Suppose you are tossing an apple up to a friend on a third-story balcony. After t seconds, the height of the apple in feet is given by $h = -16t^2 + 38.4t + 0.96$. Your friend catches the apple just as it reaches its highest point. How long does the apple take to reach your friend, and at what height above the ground does your friend catch it?

17. The barber's profit p each week depends on his charge c per haircut. It is modeled by the equation $p = -200c^2 + 2400c - 4700$. Sketch the graph of the equation. What price should he charge for the largest profit?

18. A skating rink manager finds that revenue R based on an hourly fee F for skating is represented by the function $R = -480F^2 + 3120F$. What hourly fee will produce maximum revenues?

19. The path of a baseball after it has been hit is modeled by the function $h = -0.0032d^2 + d + 3$, where h is the height in feet of the baseball and d is the distance in feet the baseball is from home plate. What is the maximum height reached by the baseball? How far is the baseball from home plate when it reaches its maximum height?

20. A lighting fixture manufacturer has daily production costs of $C = 0.25n^2 - 10n + 800$, where C is the total daily cost in dollars and n is the number of light fixtures produced. How many fixtures should be produced to yield a minimum cost?

Graph each function. Label the vertex and the axis of symmetry.

21. $y = x^2 - 2x - 3$
22. $y = 2x - \frac{1}{4}x^2$
23. $y = x^2 + 6x + 7$
24. $y = x^2 + 2x - 6$
25. $y = x^2 - 8x$
26. $y = 2x^2 + 12x + 5$
27. $y = -3x^2 - 6x + 5$
28. $y = -2x^2 + 3$
29. $y = x^2 - 6$

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Reteaching 5-3

Translating Parabolas

OBJECTIVE: Writing equations in vertex and standard forms

MATERIALS: None

- Standard form of a quadratic function is $y = ax^2 + bx + c$.
  Vertex form of a quadratic function is $y = a(x - h)^2 + k$.
- For a parabola in vertex form, the coordinates of the vertex are (h, k).

Example

Write $y = 3x^2 - 24x + 50$ in vertex form.

$y = ax^2 + bx + c$

$y = 3x^2 - 24x + 50$ ← **Verify that the equation is in standard form.**

$b = -24, a = 3$ ← **Find b and a.**

x-coordinate $= -\left(\frac{-24}{2(3)}\right)$ ← **For an equation in standard form, the x-coordinate of the vertex can be found by using $x = -\frac{b}{2a}$. Substitute.**

$= 4$ ← **Simplify.**

y-coordinate $= 3(4)^2 - 24(4) + 50$ ← **Substitute 4 into the standard form to find the y-coordinate.**

$= 2$ ← **Simplify.**

$y = 3(x - 4)^2 + 2$ ← **Substitute (4, 2) for (h, k) into the vertex form.**

Once the conversion to vertex form is complete, check by multiplying.

$y = 3(x^2 - 8x + 16) + 2$

$y = 3x^2 - 24x + 50$

The result should be the standard form of the equation.

Exercises

Write each function in vertex form. Check.

1. $y = x^2 - 2x - 3$ **2.** $y = -x^2 + 4x + 6$ **3.** $y = x^2 + 3x - 10$

4. $y = x^2 - 9x$ **5.** $y = x^2 + x$ **6.** $y = x^2 + 5x + 4$

7. $y = 4x^2 + 8x - 3$ **8.** $y = \frac{3}{4}x^2 + 9x$ **9.** $y = -2x^2 + 2x + 1$

Write each function in standard form.

10. $y = (x - 3)^2 + 1$ **11.** $y = 2(x - 1)^2 - 3$ **12.** $y = -3(x + 4)^2 + 1$

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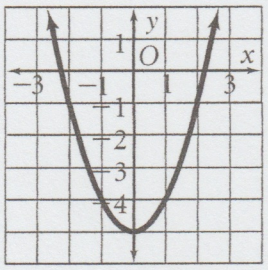
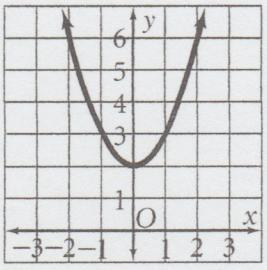
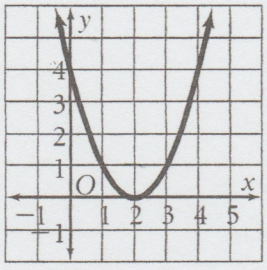
Practice 5-3

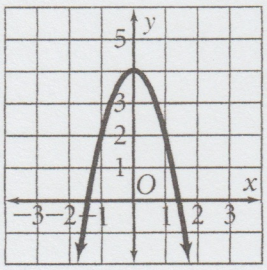
Translating Parabolas

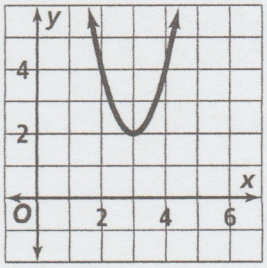
Write the equation of the parabola in vertex form.

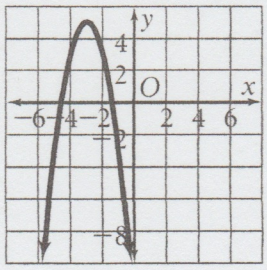
1. 

2. 

3. 

4. 

5. 

6. 

Graph each function.

7. $y = (x - 2)^2 - 3$
8. $y = (x - 6)^2 + 6$
9. $y = \frac{1}{2}(x - 1)^2 - 1$
10. $y = 8(x + 1)^2 - 2$
11. $y = -3(x - 1)^2 + 3$
12. $y = 3(x + 2)^2 + 4$
13. $y = \frac{1}{8}(x + 1)^2 - 1$
14. $y = \frac{1}{2}(x + 6)^2 - 2$
15. $y = 2(x + 3)^2 - 3$
16. $y = 4(x - 2)^2$
17. $y = -2(x + 1)^2 - 5$
18. $y = 4(x - 1)^2 - 2$

Write each function in vertex form.

19. $y = x^2 + 4x$
20. $y = 2x^2 + 8x + 3$
21. $y = -2x^2 - 8x$
22. $y = -x^2 + 4x + 4$
23. $y = x^2 - 4x - 4$
24. $y = x^2 + 5x$
25. $y = 2x^2 - 6$
26. $y = -3x^2 - x - 8$
27. $y = x^2 + 7x + 1$
28. $y = x^2 + 8x + 3$
29. $y = 2x^2 + 6x + 10$
30. $y = x^2 + 4x - 3$

Identify the vertex and the *y*-intercept of the graph of each function.

31. $y = 3(x - 2)^2 - 4$
32. $y = -\frac{1}{3}(x + 6)^2 + 5$
33. $y = 2(x - 1)^2 - 1$
34. $y = \frac{2}{3}(x + 4)^2 - 3$
35. $y = (x - 1)^2 + 2$
36. $y = -3(x - 2)^2 + 4$
37. $y = 4(x - 5)^2 + 1$
38. $y = -2(x + 5)^2 - 3$
39. $y = -5(x + 2)^2 + 5$

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Reteaching 5-4

Factoring Quadratic Expressions

OBJECTIVE: Factoring quadratic expressions **MATERIALS:** None

Example

Factor the expression $6x^2 - 5x - 4$.

$a = 6$, $b = -5$, and $c = -4$ ← **Find *a*, *b*, and *c*; they are the coefficients of each term.**

$ac = -24$ and $b = -5$ ← **We are looking for factors with product *ac* and sum *b*.**

Factors of −24	1, −24	−1, 24	2, −12	−2, 12	3, −8	−3, 8	4, −6	−4, 6
Sum of factors	−23	23	−10	10	−5	5	−2	2

The factors 3 and −8 are the combination whose sum is −5.

$\underbrace{6x^2 + 3x}\ \underbrace{- 8x - 4}$ ← **Rewrite the middle term using the factors you found.**

$3x(2x + 1) - 4(2x + 1)$ ← **Find common factors by grouping the terms in pairs.**

$(3x - 4)(2x + 1)$ ← **Rewrite using the Distributive Property.**

Check: $(3x - 4)(2x + 1)$ ← **You can check your answer by multiplying it back together.**

$6x^2 + 3x - 8x - 4$

$6x^2 - 5x - 4$

Remember that not all quadratic expressions are factorable.

Exercises

Factor each expression.

1. $x^2 + 6x + 8$

2. $x^2 - 4x + 3$

3. $2x^2 - 6x + 4$

4. $2x^2 - 11x + 5$

5. $2x^2 - 7x - 4$

6. $4x^2 + 16x + 15$

7. $x^2 - 5x - 14$

8. $7x^2 - 19x - 6$

9. $x^2 - x - 72$

10. $2x^2 + 9x + 7$

11. $x^2 + 12x + 32$

12. $4x^2 - 28x + 49$

13. $x^2 - 3x - 10$

14. $2x^2 + 9x + 4$

15. $9x^2 - 6x + 1$

16. $x^2 - 10x + 9$

17. $x^2 + 4x - 12$

18. $x^2 + 7x + 10$

19. $x^2 - 8x + 12$

20. $2x^2 - 5x - 3$

21. $x^2 - 6x + 5$

22. $3x^2 + 2x - 8$

23. $2x^2 + 11x + 5$

24. $x^2 + 3x - 28$

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Practice 5-4

Factoring Quadratic Expressions

Factor each expression completely.

1. $x^2 + 4x + 4$
2. $x^2 - 7x + 10$
3. $x^2 + 7x - 8$
4. $x^2 - 6x$
5. $2x^2 - 9x + 4$
6. $x^2 + 2x - 35$
7. $x^2 + 6x + 5$
8. $x^2 - 9$
9. $x^2 - 13x - 48$
10. $x^2 - 4$
11. $4x^2 + x$
12. $x^2 - 29x + 100$
13. $x^2 - x - 6$
14. $9x^2 - 1$
15. $3x^2 - 2x$
16. $x^2 - 64$
17. $x^2 - 25$
18. $x^2 - 81$
19. $x^2 - 36$
20. $x^2 - 100$
21. $x^2 - 1$
22. $4x^2 - 1$
23. $4x^2 - 36$
24. $9x^2 - 4$
25. $x^2 - 7x - 8$
26. $x^2 + 13x + 36$
27. $x^2 - 5x + 6$
28. $x^2 + 5x + 4$
29. $x^2 - 21x - 22$
30. $x^2 + 13x + 40$
31. $2x^2 - 5x - 3$
32. $x^2 + 10x - 11$
33. $x^2 - 14x + 24$
34. $5x^2 + 4x - 12$
35. $2x^2 - 5x - 7$
36. $2x^2 + 13x + 15$
37. $3x^2 - 7x - 6$
38. $3x^2 + 16x + 21$
39. $x^2 + 5x - 24$
40. $x^2 + 34x - 72$
41. $x^2 - 11x$
42. $3x^2 + 21x$
43. $x^2 + 8x + 12$
44. $x^2 - 10x + 24$
45. $x^2 + 7x - 30$
46. $x^2 - 2x - 168$
47. $x^2 - x - 72$
48. $4x^2 - 25$
49. $x^2 - 121$
50. $x^2 + 17x + 16$
51. $10x^2 - 17x + 3$
52. $4x^2 + 12x + 9$
53. $4x^2 - 4x - 15$
54. $9x^2 - 4$
55. $x^2 + 6x - 40$
56. $2x^2 - 8$
57. $x^2 + 18x + 77$
58. $2x^2 - 98$
59. $x^2 + 21x + 98$
60. $x^2 + 20x + 84$
61. $9x^2 + 30x + 16$
62. $8x^2 - 6x - 27$
63. $x^2 - 3x - 54$
64. $x^2 - 169$
65. $25x^2 - 9$
66. $7x^2 + 49$
67. $2x^2 - 10x - 28$
68. $x^2 + 8x + 12$
69. $x^2 - 2x - 35$
70. $x^2 + 2x - 63$
71. $20x^2 - 11x - 3$
72. $12x^2 + 4x - 5$
73. $4x^2 - 5x - 6$
74. $8x^2 + 22x - 21$
75. $3x^2 - 3x - 168$

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Reteaching 5-5

OBJECTIVE: Solving quadratic equations by graphing and factoring

MATERIALS: None

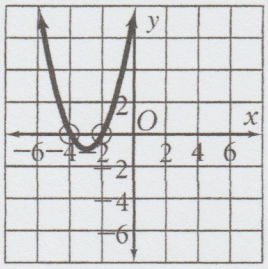
When graphing a quadratic equation, remember to use the formula $h = -\frac{b}{2a}$ to find the x-coordinate of the vertex of a parabola.
To complete the graph, plot the y-intercept $(0, c)$ and then make the parabola symmetrical.

Example

Solve the quadratic equation $x^2 + 6x + 8 = 0$ by graphing and factoring.

Graphing

Step 1
Graph the associated function $y = x^2 + 6x + 8$.



Step 2
Circle the place(s) where the graph crosses the x-axis.

Step 3
Find the values of x for the circled points.
$x = -4$ or $x = -2$

The values for x are the same for each method.

Factoring

Step 1
Factor the equation.
$(x + 4)(x + 2) = 0$

Step 2
Solve each factor for x.
$x + 4 = 0$ or $x + 2 = 0$
$x = -4$ or $x = -2$

Exercises

Solve each quadratic equation first by graphing and then by factoring.

1. $x^2 + 7x + 10 = 0$
2. $x^2 - 5x + 6 = 0$
3. $x^2 + 6x + 5 = 0$
4. $x^2 + 4x + 3 = 0$
5. $3x^2 + 10x + 3 = 0$
6. $0 = 2x^2 - 3x + 1$

Solve each quadratic equation by factoring.

7. $x^2 - 7x + 12 = 0$
8. $2x^2 + x - 15 = 0$
9. $x^2 + x - 2 = 0$
10. $3x^2 - 5x + 2 = 0$
11. $x^2 + 5x + 6 = 0$
12. $x^2 + x - 20 = 0$

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Practice 5-5

Quadratic Equations

Solve each equation by factoring, by taking square roots, or by graphing. When necessary, round your answer to the nearest hundredth.

1. $x^2 - 18x - 40 = 0$
2. $16x^2 = 56x$
3. $5x^2 = 15x$
4. $x^2 - 6x - 7 = 0$
5. $x^2 - 49 = 0$
6. $x^2 + 2x + 1 = 0$
7. $x^2 - 1 = 0$
8. $x^2 - 3x - 4 = 0$
9. $x^2 + 9x^2 + 20 = 0$
10. $6x^2 + 9 = -55x$
11. $(x + 5)^2 = 36$
12. $2x^2 - 3x = 0$
13. $2x^2 + x - 10 = 0$
14. $-4x^2 + 3x = -1$
15. $5x^2 - 6x + 1 = 0$
16. $3x^2 + 1 = -4x$
17. $-2x^2 + 2 = -3x$
18. $6x^2 + 1 = 5x$
19. $-2x^2 - x + 1 = 0$
20. $3x^2 + 5x = 2$
21. $x^2 - 6x = -8$
22. $x^2 + 6 = -7x$
23. $6x^2 + 18x = 0$
24. $2x^2 + 5 = 11x$
25. $3x^2 - 7x + 2 = 0$
26. $2x^2 - 3x = -1$
27. $2x^2 - x = 6$
28. $x^2 - 144 = 0$
29. $4x^2 + 2 = 6x$
30. $5x^2 + 2 = -7x$
31. $7x^2 + 6x - 1 = 0$
32. $2x^2 - 6x = -4$
33. $11x^2 - 12x + 1 = 0$
34. $7x^2 + 1 = -8x$
35. $x^2 + 9 = -10x$
36. $(x - 2)^2 = 18$
37. $x^2 - 8x + 7 = 0$
38. $x^2 - 16 = 0$
39. $x^2 + 6x = -8$
40. $x^2 + 3 = 4x$
41. $2x^2 + 6 = -7x$
42. $6x^2 + 2 = 7x$
43. $(x + 7)^2 = \frac{49}{16}$
44. $9x^2 - 8x = 1$
45. $10x^2 + 7x + 1 = 0$
46. $4x^2 + 2 = -9x$
47. $3x^2 + 4 = 8x$
48. $4x^2 + 5 + 9x = 0$
49. $9x^2 + 10x = -1$
50. $2x^2 + 9x + 4 = 0$
51. $2x^2 + 6x = -4$
52. $11x^2 - 1 = -10x$
53. $4x^2 = 1$
54. $6x^2 = 12x$
55. $25x^2 - 9 = 0$
56. $2x^2 + 11x = 6$
57. $8x^2 - 6x + 1 = 0$
58. $x^2 + 11 = -12x$
59. $6x^2 + 2 = 13x$
60. $x^2 = 121$
61. $4x^2 - 11x = 3$
62. $8x^2 + 6x + 1 = 0$
63. $x^2 + 9x + 8 = 0$
64. $x^2 + 8x = -12$
65. $x^2 + 6x = 40$
66. $2x^2 = 8$
67. $x^2 = x + 6$
68. $x^2 + 2x - 6 = 0$
69. $x^2 - 12 = 0$
70. $3x^2 + 4x = 6$
71. $7x^2 - 105 = 0$
72. $16x^2 = 81$
73. $x^2 + 5x + 4 = 0$
74. $x^2 + 36 = -13x$
75. $x^2 + 6 = 5x$

Name _____ Class _____ Date _____

Reteaching 5-6

Complex Numbers

OBJECTIVE: Adding, subtracting, and multiplying complex numbers

MATERIALS: None

- A *complex number* consists of a real part and an imaginary part. It is written in the form $a + bi$, where a and b are real numbers.
- When adding or subtracting complex numbers, you combine the real parts and then combine the imaginary parts.
- When multiplying complex numbers, use the Distributive Property.
- $i^2 = (\sqrt{-1})(\sqrt{-1}) = -1$ and $i = \sqrt{-1}$

Examples

Simplify $(3 - i) + (2 + 3i)$.

$(3 - i) + (2 + 3i)$

$= \textcircled{3} - \boxed{i} + \textcircled{2} + \boxed{3i}$ ← **Circle real parts. Put a square around imaginary parts.**

$= (3 + 2) + (-1 + 3)i$ ← **Combine.**

$= 5 + 2i$

Simplify $(3 + 4i)(5 + 2i)$.

$(3 + 4i)(5 + 2i)$

$= 3(5) + 3(2i) + 4i(5) + 4i(2i)$ ← **Use the Distributive Property.**

$= 15 + 6i + 20i + 8i^2$ ← **Combine real parts and imaginary parts.**

$= 15 + 26i + 8(-1)$ ← **Substitute $i^2 = -1$.**

$= 7 + 26i$

Exercises

Simplify each expression.

1. $2i + (-4 - 2i)$

2. $5i \cdot 12i$

3. $(2 + i)(2 - i)$

4. $(3 + i)(2 + i)$

5. $(4 + 3i)(1 + 2i)$

6. $3i(1 - 2i)$

7. $(6i)(-4i)$

8. $3i(4 - i)$

9. $3 - (-2 + 3i) + (-5 + i)$

10. $4i(6 - 2i)$

11. $2i + (3i)^2$

12. $(5 + 6i) + (-2 + 4i)$

13. $-14i(-4)$

14. $3i\sqrt{-6}$

15. $9(11 + 5i)$

Name _____ Class _____ Date _____

Practice 5-6

Complex Numbers

Find the first three output values for each function. Use $z = 0$ for the first input value.

1. $f(z) = z^2 + 2i$
2. $f(z) = z^2 + 1 + i$

Find the additive inverse of each of the following.

3. $2 + 3i$
4. $-4 + i$
5. $2i$
6. $-1 - i$
7. $-6i$
8. $5 - 2i$
9. $-2 + 3i$
10. 4

Find each absolute value.

11. $|-2i|$
12. $|5 + 12i|$
13. $|-1 - i|$
14. $|2 + i|$
15. $|4 + 3i|$
16. $|5 - 2i|$
17. $|3 - 2i|$
18. $|-2 + i|$
19. $|3 - 3i|$
20. $|3i|$
21. $|2i|$
22. $|4 + i|$
23. $|6 - 3i|$
24. $|-3 + i|$
25. $|4|$

Simplify each expression.

26. $\sqrt{40}$
27. $\sqrt{-88}$
28. $-\sqrt{-36}$
29. $(1 + 5i) + (1 - 5i)$
30. $(3 + 2i) - (3 + 2i)$
31. $4 - \sqrt{-25}$
32. $(2 + 6i) - (7 + 9i)$
33. $(1 + 5i)(1 - 5i)$
34. $(1 + 5i)(6 - 3i)$
35. $(5 - 6i)(6 - 2i)$
36. $(3 + 4i)(3 + 4i)$
37. $(2 + 3i)(2 - 3i)$
38. $(2 + 2i)(2 - 2i)$
39. $(-3 - 2i)(1 - 3i)$
40. $(3 + 3i) - (4 - 3i)$
41. $\sqrt{-48}$
42. $\sqrt{-300}$
43. $\sqrt{-75}$
44. $\sqrt{-16} + 2$
45. $(4 - i)(4 - i)$
46. $(4 + 2i)(1 - 7i)$
47. $(1 + 3i)(1 - 7i)$
48. $(2 + 4i)(-3 - 2i)$
49. $(11 - 12i)(11 + 12i)$
50. $(2 + 3i) + (-4 + 5i)$
51. $(5 + 14i) - (10 - 2i)$
52. $(5 + 12i)(5 - 12i)$
53. $(3 + 4i)(1 - 2i)$
54. $(6 + 2i)(1 - 2i)$
55. $(5 - 13i)(5 - 13i)$
56. $\sqrt{-44}$
57. $-\sqrt{-63}$
58. $\sqrt{-8}$
59. $(2 + 3i)(4 + 5i)$
60. $(5 + 4i) - (-1 - 2i)$
61. $(1 + 2i)(-1 - 2i)$
62. $(-1 + 4i)(1 - 2i)$
63. $(6 + 2i) + (1 - 2i)$
64. $(3 + 2i)(3 + 2i)$
65. $(-2 + 3i) + (4 + 5i)$
66. $(5 + 4i)(1 + 2i)$
67. $(-1 - 5i)(-1 + 5i)$

Solve each equation.

68. $x^2 + 80 = 0$
69. $5x^2 + 500 = 0$
70. $2x^2 + 40 = 0$
71. $3x^2 + 36 = 0$
72. $3x^2 + 75 = 0$
73. $2x^2 + 144 = 0$
74. $4x^2 + 1600 = 0$
75. $4x^2 + 1 = 0$
76. $2x^2 + 10 = 0$
77. $4x^2 + 100 = 0$
78. $x^2 + 9 = 0$
79. $9x^2 + 90 = 0$

Name _____ Class _____ Date _____

Reteaching 5-7

Completing the Square

OBJECTIVE: Solving quadratic equations by completing the square

MATERIALS: None

- Perfect square trinomials are equations in the form $x^2 + 2kx + k^2$, which can be factored into $(x + k)^2$. Completing the square produces a perfect square trinomial.
- To complete the square, you must write the equation so that the coefficient of the x^2 term equals 1.

Example

Complete the square to solve the quadratic equation.

$2x^2 + 20x - 22 = 0$

$2x^2 + 20x - 22 = 0$ ← **Look to see whether the x^2 coefficient is 1.**

$x^2 + 10x - 11 = 0$ ← **Divide each side by 2 to eliminate the x^2 coefficient.**

$x^2 + \textcircled{10}x - 11 = 0$ ← **Circle the coefficient of the linear term.**

$x^2 + \textcircled{10}x = 11$ ← **Add 11 to each side to get the variables on one side of the equal sign.**

$x^2 + \textcircled{10}x + 25 = 11 + 25$ ← **Divide the circled number by 2, square it, and add the result to each side.**

$(x + 5)^2 = 36$ ← **Factor the perfect square trinomial.**

$x + 5 = \pm 6$ ← **Take the square root of each side.**

$x = -5 \pm 6$ ← **Solve for x.**

$x = 1$ and -11 ← **Simplify.**

Exercises

Solve each equation by completing the square.

1. $x^2 + 4x = 21$

2. $x^2 - 8x = 33$

3. $x^2 + 10x = -5$

4. $3x^2 + 10x + 3 = 0$

5. $3x^2 + 4x = 3$

6. $x^2 - 5x - 5 = 0$

7. $x^2 + 7x = 0$

8. $2x^2 - 7x - 4 = 0$

9. $x^2 - x - 7 = 0$

10. $x^2 - 8x + 4 = 0$

11. $x^2 - 6x + 6 = 0$

12. $x^2 + 2x = 15$

13. $x^2 + 2x - 5 = 0$

14. $2x^2 + 8x - 10 = 0$

15. $4x^2 + 4x = 3$

Name _____ Class _____ Date _____

Practice 5-7

Completing the Square

Complete the square.

1. $x^2 + 6x + \blacksquare$
2. $x^2 - 7x + \blacksquare$
3. $x^2 + 12x + \blacksquare$
4. $x^2 + 3x + \blacksquare$
5. $x^2 - 8x + \blacksquare$
6. $x^2 + 16x + \blacksquare$
7. $x^2 + 21x + \blacksquare$
8. $x^2 - 2x + \blacksquare$

Rewrite each equation in vertex form. Then find the vertex.

9. $y = x^2 + 4x - 6$
10. $y = x^2 - 6x + 6$
11. $y = 4x^2 + 8x - 4$
12. $y = 4x^2 + 4x + 1$
13. $y = 2x^2 + 4x - 5$
14. $y = -3x^2 - 4x - 1$
15. $y = -3x^2 + 3x - 1$
16. $y = x^2 + 2x + 1$
17. $y = -5x^2 + 10x + 1$
18. $y = -2x^2 + 4x + 3$
19. $y = x^2 + 5x + \frac{5}{4}$
20. $y = -2x^2 + 10x - 11$
21. $y = 6x^2 - 12x + 1$
22. $y = -2x^2 + 8x - 9$
23. $y = 3x^2 + 9x + 6$

Solve each quadratic equation by completing the square.

24. $x^2 + 12x + 4 = 0$
25. $x^2 - x - 5 = 0$
26. $3x^2 = -12x - 3$
27. $x^2 - x - 1 = 0$
28. $4x^2 - 8x + 1 = 0$
29. $5x^2 = 8x - 6$
30. $2x^2 - 4x - 3 = 0$
31. $x^2 + 11x = 0$
32. $x^2 = 5x + 14$
33. $2x^2 + x - 1 = 0$
34. $2x^2 + 6x - 7 = 0$
35. $2x^2 = -8x + 45$
36. $x^2 = -3x - 3$
37. $4x^2 = -2x + 1$
38. $3x^2 = -6x + 9$
39. $x^2 = 7x + 12$
40. $x^2 = 3x + 7$
41. $3x^2 = 6x - 9$
42. $x^2 = -3x + 2$
43. $x^2 = -7x - 1$
44. $4x^2 = -3x + 2$
45. $2x^2 = 4x - 5$
46. $2x^2 = 5x + 5$
47. $2x^2 = 6x + 5$
48. $x^2 = 3x$
49. $x^2 = 8x$
50. $4x^2 = -2x - 3$
51. $2x^2 = -2x + 5$
52. $2x^2 = -5x - 5$
53. $3x^2 = -5x + 1$
54. $2x^2 = 2x + 4$
55. $3x^2 = 7x + 8$
56. $2x^2 = -6x + 4$
57. $x^2 = -7x - 9$
58. $2x^2 = 5x$
59. $3x^2 = -42x$
60. $2x^2 = -4x + 5$
61. $4x^2 = -x + 5$
62. $3x^2 = -3x + 1$
63. $x^2 = 3x + 4$
64. $2x^2 = 2x + 8$
65. $3x^2 = x + 4$

Solve each equation.

66. $x^2 + 2x + 1 = 9$
67. $3x^2 - 18x + 27 = 125$
68. $x^2 - 4x + 4 = 5$
69. $x^2 + 3x + \frac{9}{4} = \frac{13}{4}$
70. $x^2 + 3x + \frac{9}{4} = -\frac{15}{4}$
71. $x^2 + 3x + \frac{9}{4} = \frac{41}{4}$
72. $x^2 + 7x + \frac{49}{4} = \frac{53}{4}$
73. $x^2 + 3x + \frac{9}{4} = \frac{29}{4}$
74. $x^2 - 6x + 9 = 7$

Name _____ Class _____ Date _____

Reteaching 5-8

The Quadratic Formula

OBJECTIVE: Solving quadratic equations by using the Quadratic Formula

MATERIALS: None

Follow each step below to solve any quadratic equation by using the Quadratic Formula.

1. Write the equation in the standard form $ax^2 + bx + c = 0$.
2. Substitute a-, b-, and c-values into the Quadratic Formula.
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
3. Simplify. Use imaginary numbers if necessary.
4. Check the solution(s) by substituting the values into the original equation.

Example

Use the Quadratic Formula to solve $x^2 + 2 = -2x$. Check your solution.

$$x^2 + 2 = -2x$$

$x^2 + 2x + 2 = 0$ ← **Write in standard form.**

$\underline{1}x^2 + \textcircled{2}x + \boxed{2} = 0$ ← **Underline a, circle b, and put a square around c.**

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$ ← **Substitute 1 for a, 2 for b, and 2 for c into the Quadratic Formula.**

$$= \frac{-2 \pm \sqrt{-4}}{2}$$ ← **Simplify to find the values of x.**

$$= \frac{-2 \pm 2i}{2}$$

$$= -1 \pm i$$

Check:

$x^2 + 2 = -2x$

$(-1 + i)^2 + 2 \stackrel{?}{=} -2(-1 + i)$

$1 - 2i + i^2 + 2 \stackrel{?}{=} 2 - 2i$

$1 - 2i - 1 + 2 \stackrel{?}{=} 2 - 2i$

$2 - 2i = 2 - 2i$✓

$x^2 + 2 = -2x$

$(-1 - i)^2 + 2 \stackrel{?}{=} -2(-1 - i)$

$1 + 2i + i^2 + 2 \stackrel{?}{=} 2 + 2i$

$1 + 2i - 1 + 2 \stackrel{?}{=} 2 + 2i$

$2 + 2i = 2 + 2i$✓

Exercises

Solve each equation using the Quadratic Formula.

1. $x^2 - 3x + 2 = 0$
2. $-x^2 + 5x = 9$
3. $10x - 6 = 5x^2$
4. $x + 2x^2 + 1 = -1 - x$
5. $2x^2 + x = 10$
6. $2x + 1 = 2x^2$

Name _____ Class _____ Date _____

Practice 5-8

The Quadratic Formula

Evaluate the discriminant of each equation. Tell how many solutions each equation has and whether the solutions are real or imaginary.

1. $y = x^2 + 10x - 25$
2. $y = x^2 + 10x + 10$
3. $y = 9x^2 - 24x$
4. $y = 4x^2 - 4x + 1$
5. $y = 4x^2 - 5x + 1$
6. $y = 4x^2 - 3x + 1$
7. $y = x^2 + 3x + 4$
8. $y = x^2 + 7x - 3$
9. $y = -2x^2 + 3x - 5$
10. $y = x^2 - 5x + 4$
11. $y = x^2 + 12x + 36$
12. $y = x^2 + 2x + 3$
13. $y = 2x^2 - 13x - 7$
14. $y = -5x^2 + 6x - 4$
15. $y = -4x^2 - 4x - 1$

Solve each equation using the Quadratic Formula.

16. $x^2 + 6x + 9 = 0$
17. $x^2 - 15x + 56 = 0$
18. $3x^2 - 5x + 2 = 0$
19. $2x^2 + 3x + 5 = 0$
20. $10x^2 - 23x + 12 = 0$
21. $4x^2 + x - 5 = 0$
22. $x^2 + 8x + 15 = 0$
23. $3x^2 + 2x + 1 = 0$
24. $4x^2 + x + 5 = 0$
25. $x^2 - 4x - 12 = 0$
26. $x^2 = 3x + 2$
27. $2x^2 - 5x + 2 = 0$
28. $x^2 + 6x - 4 = 0$
29. $x^2 = 2x - 5$
30. $3x^2 + 7 = -6x$
31. $2x^2 + 6x + 3 = 0$
32. $x^2 = -18x - 80$
33. $x^2 + 9x - 13 = 0$
34. $x^2 - 8x + 25 = 0$
35. $4x^2 + 13x = 12$
36. $3x^2 - 5x = -12$
37. $3x^2 + 4x + 5 = 0$
38. $2x^2 = 3x - 7$
39. $5x^2 + 2x + 1 = 0$
40. $5x^2 + x + 3 = 0$
41. $5x^2 + x = 3$
42. $5x^2 - 2x + 7 = 0$
43. $x^2 - 2x + 3 = 0$
44. $-2x^2 + 3x = 24$
45. $4x^2 = 5x - 6$
46. $x^2 + 6x + 5 = 0$
47. $x^2 - 6x = -8$
48. $x^2 - 6x = -6$

Solve.

49. A model of the daily profits p of a gas station based on the price per gallon g is $p = -15{,}000g^2 + 34{,}500g - 16{,}800$. Use the discriminant to find whether the station can profit \$4000 per day. Explain.

Solve each equation using the Quadratic Formula. Find the exact solutions. Then approximate any radical solutions. Round to the nearest hundredth.

50. $x^2 - 2x - 3 = 0$
51. $x^2 + 5x + 4 = 0$
52. $x^2 - 2x - 8 = 0$
53. $7x^2 - 12x + 3 = 0$
54. $5x^2 + 5x - 1 = 0$
55. $4x^2 + 5x + 1 = 0$
56. $6x^2 + 5x - 4 = 0$
57. $x^2 + x = 6$
58. $x^2 - 13x = 48$
59. $2x^2 + 5x = 0$
60. $x^2 + 3x - 3 = 0$
61. $x^2 - 4x + 1 = 0$
62. $9x^2 - 6x - 7 = 0$
63. $x^2 - 35 = 2x$
64. $x^2 + 7x + 10 = 0$