Properties of Parallel Lines

OBJECTIVE: Relating the measures of angles formed by parallel lines and a transversal

MATERIALS: Ruler, protractor

Example

If $m \angle 1 = 100$, find the measure of each of the other seven angles.

$$m \angle 1 + m \angle 2 = 180; m \angle 2 = 80$$

$$m \angle 1 + m \angle 4 = 180; m \angle 4 = 80$$

$$\angle 1 \cong \angle 3; m \angle 3 = 100$$

$$\angle 3 \cong \angle 5; m \angle 5 = 100$$

$$m \angle 3 + m \angle 8 = 180; m \angle 8 = 80$$

$$\angle 3 \cong \angle 7; m \angle 7 = 100$$

$$m \angle 6 + m \angle 7 = 180; m \angle 6 = 80$$

Supplementary angles

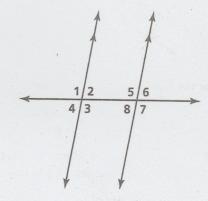
Vertical angles

Alternate interior angles

Same-side interior angles

Corresponding angles

Supplementary angles



Exercises

Complete the following to find measures of angles associated with a pair of parallel lines and a transversal.

- **1. a.** Draw a pair of parallel lines using lined paper or the edges of a ruler. Then draw a transversal that intersects the two parallel lines.
 - **b.** Use a protractor to measure one of the angles formed. Record the measure on your drawing.
 - c. Find the measures of the other seven angles without measuring.
 - d. Verify the angle measures by measuring each with a protractor.

Find the measure of each angle in the diagram at the right.

2. *m*∠1

3. *m*∠2

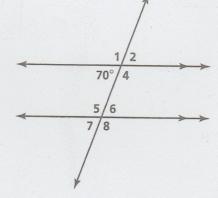
4. m 4

5. *m*∠5

6. m \(\) 6

7. *m*∠7

8. *m*∠8



Lesson 3-1 Reteaching

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Properties of Parallel Lines

Classify each pair of angles as alternate interior angles, same-side interior angles, or corresponding angles.

1.



2



3.



4.



E

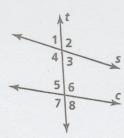


6



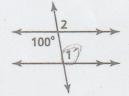
Use the figure on the right to answer Exercises 7-9.

- **7.** Name all pairs of corresponding angles formed by the transversal *t* and lines *s* and *c*.
- **8.** Name all pairs of alternate interior angles formed by the transversal *t* and lines *s* and *c*.
- **9.** Name all pairs of same-side interior angles formed by the transversal *t* and lines *s* and *c*.

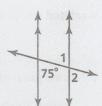


Find $m \angle 1$ and then $m \angle 2$. Justify each answer.

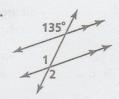
10.



11

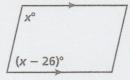


12.

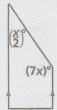


Algebra Find the value of x. Then find the measure of each angle.

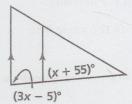
12



14



15.



16. Developing Proof Supply the missing reasons in this two-column proof.

Given: a | b

Prove: $\angle 1 \cong \angle 3$

Statements

1. a || b

2. ∠1 ≅ ∠2

3. ∠2 ≅ ∠3

4. ∠1 ≅ ∠3

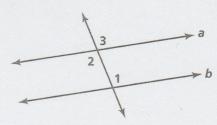
Reasons

1. Given

a. ?

b. ?

c. ?



Proving Lines Parallel

OBJECTIVE: Writing flow proofs

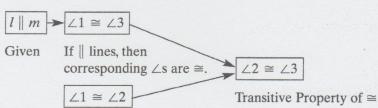
MATERIALS: None

Example

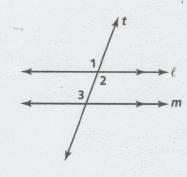
Write a flow proof for Theorem 3-1: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

Given: $l \parallel m$

Prove: $\angle 2 \cong \angle 3$



Vertical angles are ≅.



Exercises

1. Complete the flow proof for Theorem 3-2 using the following steps. Then write the reasons for each step.

a.
$$\angle 2$$
 and $\angle 3$ are supplementary. **b.** $\angle 1 \cong \angle 3$

b.
$$\angle 1 \cong \angle 3$$

d.
$$m \angle 1 + m \angle 2 = 180$$

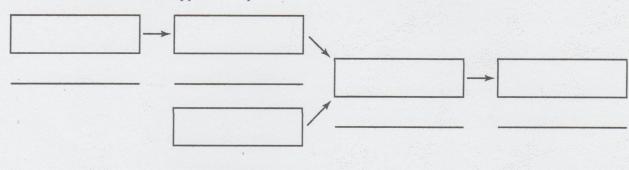
e.
$$m \angle 3 + m \angle 2 = 180$$

Theorem 3-2: If two parallel lines are cut by a transversal, then sameside interior angles are supplementary.



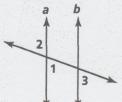
$$l \parallel m$$

 $\angle 2$ and $\angle 3$ are supplementary.



2. Write a flow proof for the following:

$$a \parallel b$$



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Proving Lines Parallel

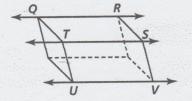
1. Developing Proof Complete the paragraph proof for the figure shown.

Given: $\angle RQT$ and $\angle QTS$ are supplementary.

 $\angle TSV$ and $\angle SVU$ are supplementary.

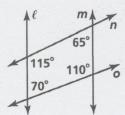
Prove: $\overrightarrow{QR} \parallel \overrightarrow{UV}$

Proof Because $\angle RQT$ and $\angle QTS$ are supplementary, $\angle RQT$ and $\angle QTS$ are **a.** ? angles. By the Same-Side Interior Angles Theorem, **b.** ? \parallel **c.** ? Because $\angle TSV$ and $\angle SVU$ are supplementary, $\angle TSV$ and $\angle SVU$ are **d.** ? angles. By the **e.** ? Theorem, $\overrightarrow{TS} \parallel \overrightarrow{UV}$. Because \overrightarrow{QR} and \overrightarrow{UV} both are parallel to **f.** ? $\overrightarrow{QR} \parallel \overrightarrow{UV}$ by Theorem **g.** ?

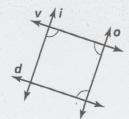


Which lines or segments are parallel? Justify your answer with a theorem or postulate.

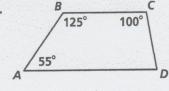
2.



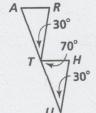
3.



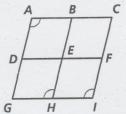
4.



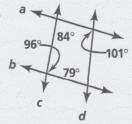
5.



6.

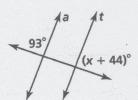


7.

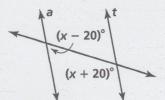


Algebra Find the value of x for which $a \parallel t$.

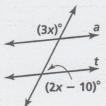
8.



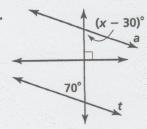
9.



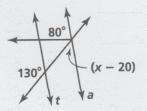
10.



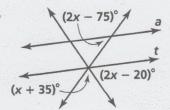
11.



12.



10



Parallel Lines and the Triangle Angle-Sum Theorem

OBJECTIVE: Classifying triangles and finding the measures of their angles

MATERIALS: Ruler

Example

In the diagram at the right, ACED has four right angles. Find the missing angle measures in $\triangle ABC$, and classify them. Then classify $\triangle ABC$ in as many ways as you can.

$$m \angle CAB + m \angle DAB = 90$$

Angle Addition Postulate

$$m \angle CAB + 30 = 90$$

Substitution

$$m \angle CAB = 60$$

Subtraction Property of Equality

$$m \angle ACB + m \angle CAB + m \angle ABC = 180$$

Triangle Angle-Sum Theorem

$$m \angle ACB + 60 + 60 = 180$$

Substitution

$$m \angle ACB + 120 = 80$$

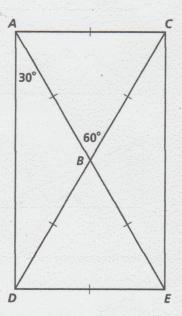
Addition

$$m \angle ACB = 60$$

Subtraction Property of Equality

Because $m \angle CAB < 90$ and $m \angle ACB < 90$, $\angle CAB$ and $\angle ACB$ are acute.

Therefore, $\triangle ABC$ is equilateral, equiangular, and acute.



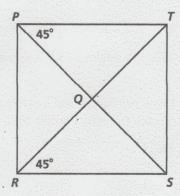
Exercises

Refer to the diagram above.

- **1.** Find the missing angle measures in $\triangle ABD$, $\triangle CBE$, and $\triangle BDE$.
- 2. Name the eight triangles in the diagram. Then sketch the triangles, and classify them in as many ways as possible. ($\triangle ABC$ has been classified in the example.)

In the diagram at the right, $\angle RPT$, $\angle PTS$, $\angle TSR$, and $\angle SRP$ are right angles.

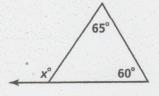
- **3.** Find the missing angle measures in $\triangle PQT$, $\triangle PQR$, $\triangle RQS$, and $\triangle SQT$.
- **4.** Measure the side lengths of $\triangle PQT$, $\triangle PQR$, $\triangle RQS$, and $\triangle SQT$ to the nearest millimeter.
- 5. List and classify each triangle. (Hint: There are eight triangles.)



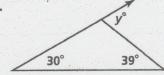
Parallel Lines and the Triangle Angle-Sum Theorem

Find the value of each variable.

1.



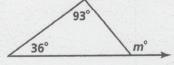
2.



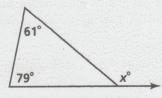
3



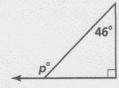
4.



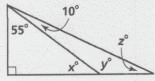
5.

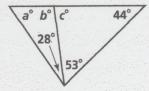


6.

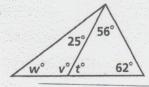


7.



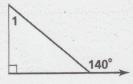


9.

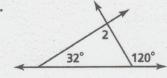


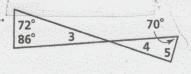
Find the measure of each numbered angle.

10.

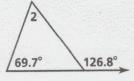


11.

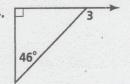




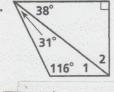
13.



14.



15.



- 16. The sides of a triangle are 10 cm, 8 cm, and 10 cm. Classify the triangle.
- 17. The angles of a triangle are 44°, 110°, and 26°. Classify the triangle.

Use a protractor and a centimeter ruler to measure the angles and the sides of each triangle. Classify each triangle by its angles and sides.

18.





20.



The Polygon Angle-Sum Theorems

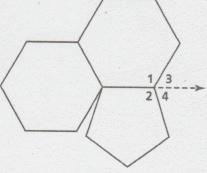
OBJECTIVE: Finding the sum of the measures of the interior and exterior angles of polygons

MATERIALS: None

Example

A pattern of regular hexagons and regular pentagons covers a soccer ball. Find the measures of an interior and an exterior angle of the hexagon and an interior and an exterior angle of the pentagon.

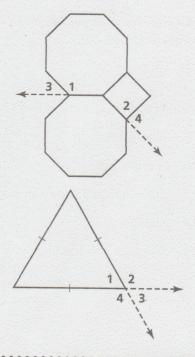
- The sum of the measures of the interior angles of a hexagon equals (n-2)180 = (6-2)180 = 720.
- $m \angle 1 = 720 \div 6 = 120$.
- The sum of the measures of the exterior angles of a hexagon equals 360.
- $m \angle 3 = 360 \div 6 = 60$.
- The sum of the measures of the interior angles of a pentagon equals (5-2)180 = 540.
- $m \angle 2 = 540 \div 5 = 108$.
- The sum of the measures of the exterior angles of a pentagon equals 360.
- $m \angle 4 = 360 \div 5 = 72$.
- An interior angle of the hexagon measures 120, and an exterior angle measures 60.
- An interior angle of the pentagon measures 108, and an exterior angle measures 72.



Exercises

Sometimes regular octagons are pieced around a square to form a quilt pattern.

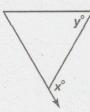
- **1.** Classify $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ as interior or exterior angles.
- **2.** Find the measures of $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.
- **3.** Classify $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ as interior angles, exterior angles, or neither.
- **4.** Find the measures of $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$.



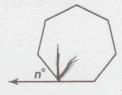
The Polygon Angle-Sum Theorems

Find the values of the variables for each polygon. Each is a regular polygon.

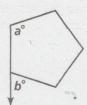
1.



2.

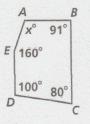


3.

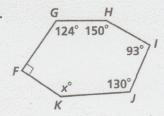


Find the missing angle measures.

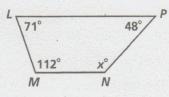
4.



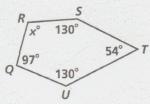
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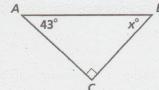
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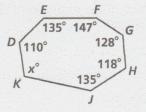
7.



8.



9.



For a regular 12-sided polygon, find each of the following.

- 10. the measure of an exterior angle
- 11. the measure of an interior angle

The measure of an interior angle of a regular polygon is given. Find the number of sides.

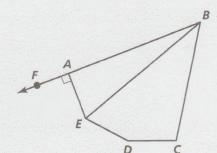
12. 120

13. 108

14. 135

Identify each item in Exercises 15-18 in the figure.

- 15. quadrilateral
- 16. exterior angle
- 17. pair of supplementary angles
- 18. pentagon
- **19.** A regular polygon has an exterior angle of measure 18. How many sides does the polygon have?



Lines in the Coordinate Plane

OBJECTIVE: Writing and graphing equations of lines

MATERIALS: Graphing paper

of lines

If you know two points on a line, or if you know one point and the slope of a line, then you can find the equation of the line.

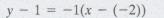
Example

Write an equation of the line that contains the points J(4, -5) and K(-2, 1). Graph the line.

If you know two points on a line, first find the slope using $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$m = \frac{1 - (-5)}{-2 - 4} = \frac{6}{-6} = -1$$

Now you know two points and the slope of the line. Select one of the points to substitute for (x_1, y_1) . Then find the equation using the point-slope form $y - y_1 = m(x - x_1)$.



$$y - 1 = -1(x + 2)$$

Simplify within parentheses. You may leave your equation in this form or further simplify to find the slope-intercept form.

$$y - 1 = -x - 2$$
$$y = -x - 1$$

Answer: Either y - 1 = -1(x + 2) or y = -x - 1 is acceptable.

Exercises

Write an equation for the line with the given slope that contains the given point. Graph each line.

1. slope
$$2, (2, -2)$$

2. slope
$$\frac{1}{3}$$
, $(-6, -2)$

3. slope
$$-1, (-3, 0)$$

4. slope
$$\frac{5}{6}$$
, $(-6, -3)$

5. slope
$$-\frac{1}{2}$$
, $(-4, 3)$

6. slope
$$0, (3, 1)$$

Write an equation for the line containing the given points. Graph each line.

8.
$$(-4,5), (3,-2)$$

9.
$$(0,1), (-5,-1)$$

11.
$$(-3,0),(-5,4)$$

12.
$$(-3,4), (-3,-1)$$

Write an equation for the line with the given information. Graph each line.

13. contains point
$$(4, -2)$$
, slope -3

14. contains points
$$(3, -1), (5, 5)$$

15. contains point
$$(2, 1)$$
, slope $\frac{1}{4}$

16. contains point
$$(8, -2)$$
, slope $-\frac{3}{4}$

17. contains points
$$(-4, 5), (-3, 4)$$

18. contains points
$$(1,1), (2,1)$$

Lines in the Coordinate Plane

Write an equation of the line with the given slope that contains the given point.

1.
$$F(3, -6)$$
, slope $\frac{1}{3}$

2.
$$Q(5,2)$$
, slope -2 **3.** $A(3,3)$, slope 7

3.
$$A(3,3)$$
, slope 7

4.
$$B(-4, -1)$$
, slope $-\frac{1}{2}$

5.
$$L(-3, -2)$$
, slope $\frac{1}{6}$ **6.** $R(15, 10)$, slope $\frac{4}{5}$ **7.** $D(1, -9)$, slope 4

6.
$$R(15, 10)$$
, slope $\frac{2}{3}$

7.
$$D(1, -9)$$
, slope 4

8.
$$W(0,6)$$
, slope -1

Graph each line using slope-intercept form.

9.
$$2y = 8x - 2$$

9.
$$2y = 8x - 2$$
 10. $2y = \frac{1}{2}x - 10$ **11.** $3x + 9y = 18$

11.
$$3x + 9y = 18$$

12.
$$-x + y = -1$$

13.
$$y + 7 = 2x$$

14.
$$4x - 2y = 6$$

14.
$$4x - 2y = 6$$
 15. $5 - y = \frac{3}{4}x$

16.
$$\frac{1}{3}x = \frac{1}{2}y - 1$$

Graph each line.

17.
$$y = 5x + 4$$

18.
$$y = \frac{1}{2}x - 3$$

19.
$$x = -2$$

20.
$$y = -2x$$

21.
$$y = -5$$

22.
$$y = x$$

23.
$$y = -\frac{2}{3}x + 2$$

24.
$$x = 2.5$$

Write an equation of the line containing the given points.

25.
$$A(2,7), B(3,4)$$

26.
$$P(-1,3), Q(0,4)$$

27.
$$S(10,2), T(2,-2)$$

27.
$$S(10,2), T(2,-2)$$
 28. $D(7,-4), E(-5,2)$

29.
$$G(-2,0), H(3,10)$$

30.
$$B(3,5), C(-6,2)$$

31.
$$X(-1, -1), Y(4, -2)$$
 32. $M(8, -3), N(7, 3)$

32.
$$M(8, -3), N(7, 3)$$

Write equations for (a) the horizontal line and (b) the vertical line that contain the given point.

33.
$$Z(2, -11)$$

35.
$$R(-4, -4)$$

36.
$$F(-1,8)$$

Graph each line using intercepts.

37.
$$3x - y = 12$$

38.
$$2x + 4y = -4$$

39.
$$\frac{1}{2}x + \frac{1}{2}y = 3$$

40.
$$12x - 3y = -6$$

41.
$$2x - 2y = 8$$

42.
$$\frac{1}{4}x + 2y = 2$$

42.
$$\frac{1}{4}x + 2y = 2$$
 43. $-6x + 1.5y = 18$ **44.** $0.2x + 0.3y = 1.8$

44.
$$0.2x + 0.3y = 1.8$$

45. The equation P = \$3.90 + \$0.10x represents the hourly pay (P)a worker receives for loading x number of boxes onto a truck.

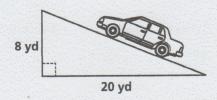
a. What is the slope of the line represented by the given equation?

b. What does the slope represent in this situation?

c. What is the y-intercept of the line?

d. What does the y-intercept represent in this situation?

46. The Blackberrys' driveway is difficult to get up in the winter ice and snow because of its slope. What is the equation of the line that represents the Blackberrys' driveway?



Slopes of Parallel and Perpendicular Lines

OBJECTIVE: Identifying and writing equations for parallel and perpendicular lines

MATERIALS: Graphing paper

Example 1

Write an equation for the line that contains G(4, -3) and is parallel to \overrightarrow{EF} : $-\frac{1}{2}x + 2y = 6$. Write another equation for the line that contains G and is perpendicular to \overrightarrow{EF} . Graph the three lines.

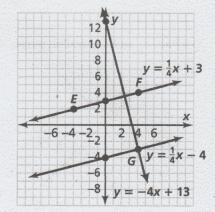
Step 1 Rewrite in slope-intercept form: $y = \frac{1}{4}x + 3$

Step 2 Use point-slope form to write an equation for each line.

Parallel line:
$$m = \frac{1}{4}$$
 Perpendicular line: $m = y - (-3) = \frac{1}{4}(x - 4)$ $y - (-3) = -4(x - 4)$ $y = -4x + 13$

Perpendicular line: m = -4

$$y - (-3) = -4(x - 4)$$
$$y = -4x + 13$$



Example 2

HII FIGHTS RESERVED.

Given points J(-1, 4), K(2, 3), L(5, 4), and M(0, -3), are \overrightarrow{JK} and \overrightarrow{LM} parallel, perpendicular, or neither?

$$-\frac{1}{3} \neq \frac{7}{5}$$
 Their slopes are not equal, so they are not parallel.

 $\frac{1}{3} \cdot \frac{7}{5} \neq -1$ The product of their slopes is not -1 , so they are not perpendicular.

Exercises

Find the slope of a line (a) parallel to and (b) perpendicular to each line.

1.
$$y = -2x$$

2.
$$y = \frac{1}{4}x - 6$$

3.
$$x = -3$$

Write an equation for the line that (a) contains G and is parallel to \overrightarrow{EF} . Write another equation for the line that (b) contains G and is perpendicular to \overrightarrow{EF} . (c) Graph the three lines to check your answers.

4.
$$\overrightarrow{EF}: y = -2x + 5, G(1, 2)$$

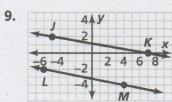
4.
$$\overrightarrow{EF}: y = -2x + 5, G(1,2)$$
 5. $\overrightarrow{EF}: 6y + 4x = -12, G(0,-4)$ **6.** $\overrightarrow{EF}: x - \frac{1}{3}y = 4, G(-3,-2)$

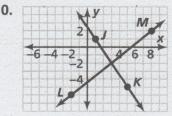
6.
$$\overrightarrow{EF}: x - \frac{1}{3}y = 4, G(-3, -2)$$

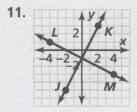
Tell whether \overrightarrow{JK} and \overrightarrow{LM} are parallel, perpendicular, or neither.

7.
$$J(2,0), K(-1,3), L(0,4), M(-1,5)$$

8.
$$J(-4, -5), K(5, 1), L(6, 0), M(4, 3)$$







12.
$$\overrightarrow{JK} : y = \frac{1}{5}x + 2$$

 $\overrightarrow{LM} : y = 5x - \frac{1}{2}$

13.
$$\overrightarrow{JK} : 2y + \frac{1}{2}x = -2$$

 $\overrightarrow{LM} : 2x + 8y = 8$

14.
$$\overrightarrow{JK}: y = -1$$
 $\overrightarrow{LM}: x = 0$

Slopes of Parallel and Perpendicular Lines

Are the lines parallel, perpendicular, or neither? Explain.

1.
$$y = 3x - 2$$

$$y = 3x - 2$$
 2. y

2.
$$y = \frac{1}{2}x + 1$$

2.
$$y = \frac{1}{2}x + 1$$
 3. $\frac{2}{3}x + y = 4$ **4.** $-x - y = -1$

$$y = \frac{1}{3}x + 2$$
 $-4y = 8x + 3$ $y = -\frac{2}{3}x + 8$ $y + x = 7$

5.
$$y = 2$$

x = 0

6.
$$3x + 6y = 30$$

 $4y + 2x = 9$

$$8y - x = 8$$

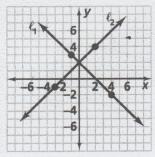
$$-x-y=-1$$

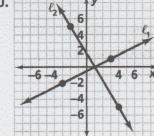
$$y + x = 7$$

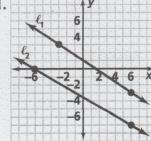
6.
$$3x + 6y = 30$$
 7. $y = x$ **8.** $\frac{1}{3}x + \frac{1}{2}y = 1$

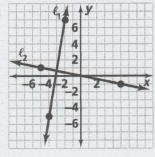
$$\frac{3}{4}y + \frac{1}{2}x = 1$$

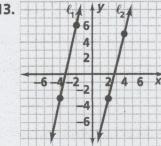
Are lines l_1 and l_2 parallel, perpendicular, or neither? Explain.

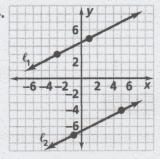












Write an equation for the line perpendicular to \overrightarrow{XY} that contains point Z.

15.
$$\overrightarrow{XY}$$
: $3x + 2y = -6$, $Z(3,2)$

16.
$$\overrightarrow{XY}$$
: $y = \frac{3}{4}x + 22$, $Z(12, 8)$

16.
$$\overrightarrow{XY}$$
: $y = \frac{3}{4}x + 22$, $Z(12, 8)$ **17.** \overrightarrow{XY} : $-x + y = 0$, $Z(-2, -1)$

Write an equation for the line parallel to \overrightarrow{XY} that contains point Z.

18.
$$\overrightarrow{XY}$$
: $6x - 10y + 5 = 0$, $Z(-5,3)$ **19.** \overrightarrow{XY} : $y = -1$, $Z(0,0)$ **20.** \overrightarrow{XY} : $x = \frac{1}{2}y + 1$, $Z(1,-2)$

19.
$$\overrightarrow{XY}$$
: $y = -1, Z(0, 0)$

20.
$$\overrightarrow{XY}$$
: $x = \frac{1}{2}y + 1$, $Z(1, -2)$

21. Two planes are flying side by side at the same altitude. It is important that their paths do not intersect. One plane is flying along the path given by the line 4x - 2y = 10. What is the slope-intercept form of the line that must be the path of another plane passing through the point L(-1, -2) so that the planes do not collide? Graph the paths of the two planes.

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