

Reteaching 4-1

Congruent Figures and Corresponding Parts

OBJECTIVE: Recognizing congruent figures and their corresponding parts

MATERIALS: None

Example

$\triangle ABC \cong \triangle XYZ$. Find $m\angle A$.

Because the triangles are congruent, all corresponding parts are congruent.

Sides: $\overline{AB} \cong \overline{XY}$, $\overline{BC} \cong \overline{YZ}$, $\overline{AC} \cong \overline{XZ}$

Angles: $\angle A \cong \angle X$, $\angle B \cong \angle Y$, $\angle C \cong \angle Z$

Because $\angle B \cong \angle Y$, $m\angle B \cong 37$.

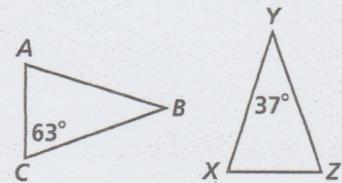
Use the Triangle Angle-Sum Theorem to find $m\angle A$.

$$m\angle A + m\angle B + m\angle C = 180$$

$$m\angle A + 37 + 63 = 180$$

$$m\angle A + 100 = 180$$

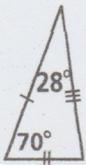
$$m\angle A = 80$$



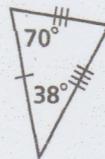
Exercises

Match each triangle in the first column with a congruent triangle in the second column.

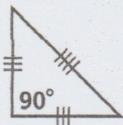
1.



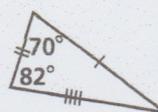
a.



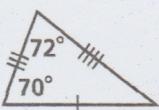
2.



b.



3.

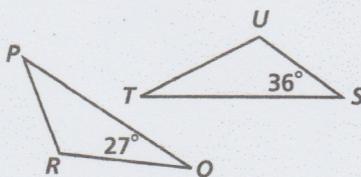


c.

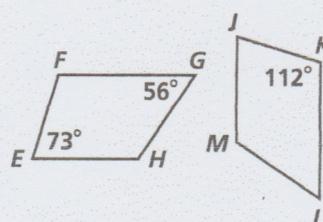


Find the measure of the indicated angle.

4. $\triangle PQR \cong \triangle STU$. Find $m\angle U$.



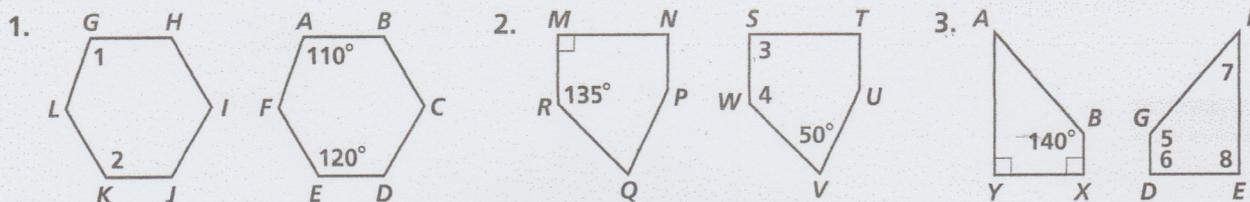
5. $EFGH \cong JKLM$. Find $m\angle M$.



Practice 4-1

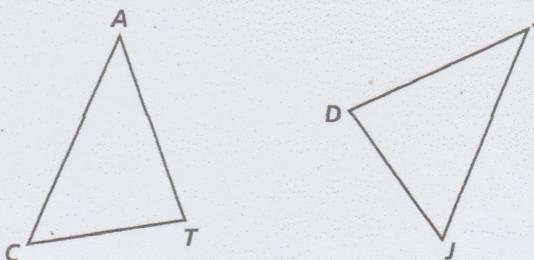
Congruent Figures and Corresponding Parts

Each pair of polygons is congruent. Find the measures of the numbered angles.



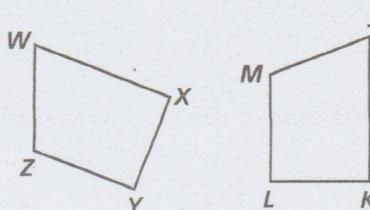
$\triangle CAT \cong \triangle JSD$. List each of the following.

4. three pairs of congruent sides
5. three pairs of congruent angles



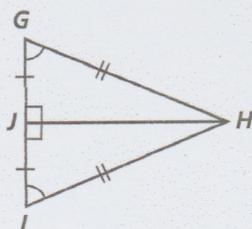
$WXYZ \cong JKLM$. List each of the following.

6. four pairs of congruent sides
7. four pairs of congruent angles

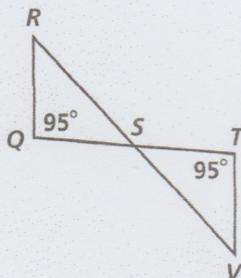


State whether the pairs of figures are congruent. Explain.

8. $\triangle GHJ$ and $\triangle IHJ$

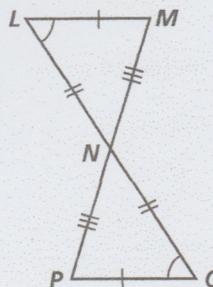


9. $\triangle QRS$ and $\triangle TVS$



10. **Developing Proof** Use the information given in the diagram. Give a reason that each statement is true.

- a. $\angle L \cong \angle Q$
- b. $\angle LNM \cong \angle PNQ$
- c. $\angle M \cong \angle P$
- d. $\overline{LM} \cong \overline{QP}$, $\overline{LN} \cong \overline{QN}$, $\overline{MN} \cong \overline{PN}$
- e. $\triangle LNM \cong \triangle QNP$



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Reteaching 4-2

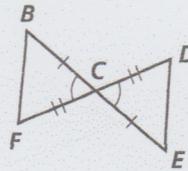
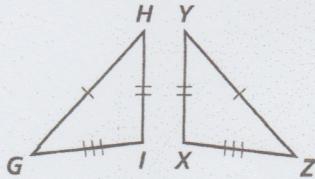
Triangles Congruent by SSS and SAS

OBJECTIVE: Proving two triangles congruent using the SSS and SAS postulates

MATERIALS: Ruler, protractor

Example

Name the triangle congruence postulate you can use to prove each pair of triangles congruent.



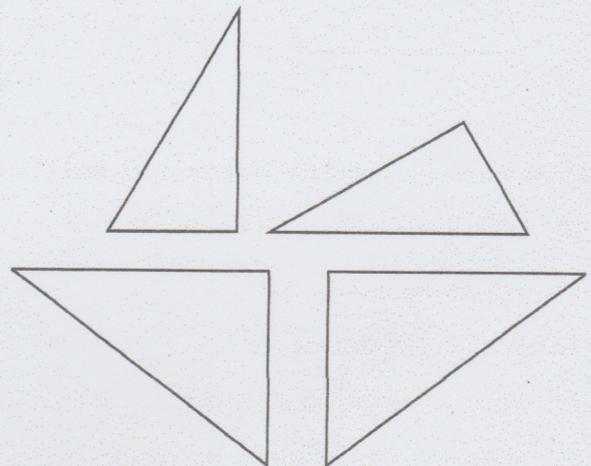
a. Because three sides of $\triangle GHI$ are congruent to three sides of $\triangle ZYX$, $\triangle GHI \cong \triangle ZYX$ by the SSS Postulate.

b. Because two sides and the included angle of $\triangle BCF$ are congruent to two sides and the included angle of $\triangle ECD$, $\triangle BCF \cong \triangle ECD$ by the SAS Postulate.

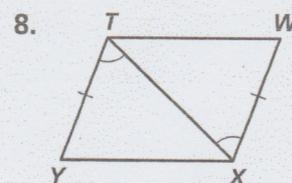
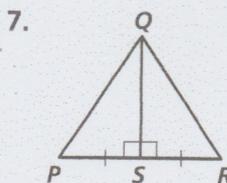
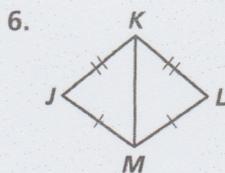
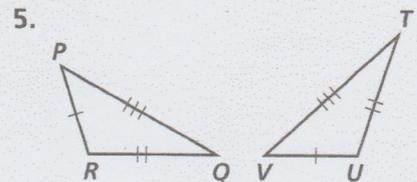
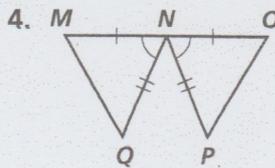
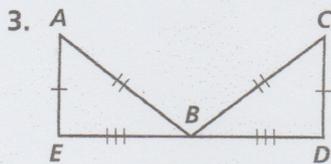
Exercises

Refer to the triangles at the right.

- Use a ruler to show that the top two triangles at the right are congruent by the SSS Postulate.
- Use a ruler and a protractor to show that the two large triangles at the right are congruent by the SAS Postulate.



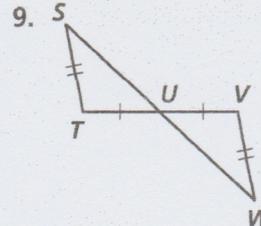
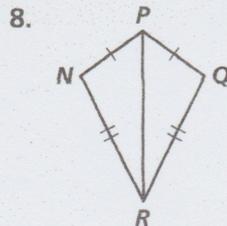
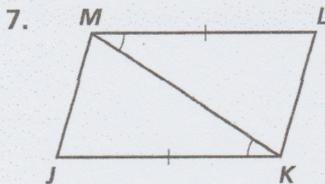
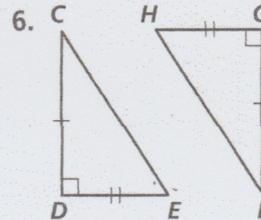
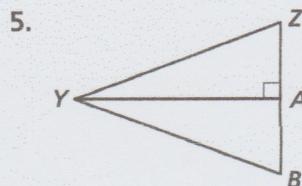
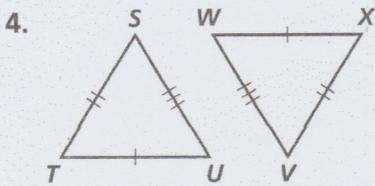
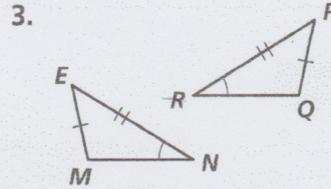
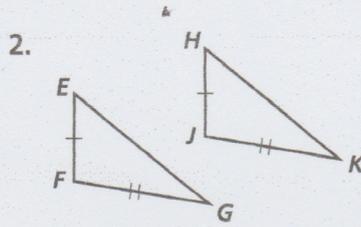
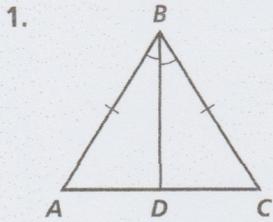
Name the triangle congruence postulate you can use to prove each pair of triangles congruent. Then state the triangle congruence.



Practice 4-2

Triangle Congruence by SSS and SAS

Decide whether you can use the SSS or SAS Postulate to prove the triangles congruent. If so, write the congruence statement, and identify the postulate. If not, write *not possible*.



Draw a triangle. Label the vertices *A*, *B*, and *C*.

- What angle is between \overline{BC} and \overline{AC} ?
- What sides include $\angle B$?
- What angles include \overline{AB} ?
- What side is included between $\angle A$ and $\angle C$?
- Developing Proof** Supply the reasons in this proof.

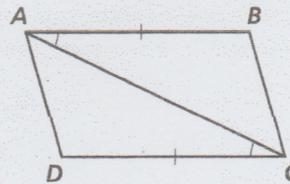
Given: $\overline{AB} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$
 Prove: $\triangle ABC \cong \triangle CDA$

Statements

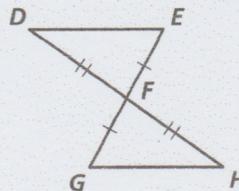
- $\overline{AB} \cong \overline{DC}$, $\angle BAC \cong \angle DCA$
- $\overline{AC} \cong \overline{CA}$
- $\triangle ABC \cong \triangle CDA$

Reasons

- ?
- ?
- ?



- Write a proof.
 Given: $\overline{EF} \cong \overline{FG}$, $\overline{DF} \cong \overline{FH}$
 Prove: $\triangle DFE \cong \triangle HFG$



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Reteaching 4-3

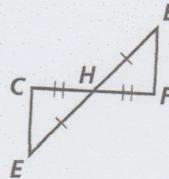
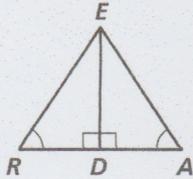
Triangle Congruence by AAS and ASA

OBJECTIVE: Proving two triangles congruent by the ASA Postulate and the AAS Theorem

MATERIALS: Ruler, protractor

Example

Tell whether the ASA Postulate or the AAS Theorem can be applied directly to prove the triangles congruent.



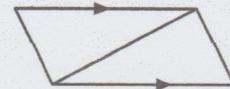
- a. Because $\angle RDE$ and $\angle ADE$ are right angles, they are congruent. $\overline{ED} \cong \overline{ED}$ by the Reflexive Property of \cong , and it is given that $\angle R \cong \angle A$. Therefore, $\triangle RDE \cong \triangle ADE$ by the AAS Theorem.

- b. It is given that $\overline{CH} \cong \overline{FH}$ and $\overline{EH} \cong \overline{BH}$. Because $\angle CHE$ and $\angle FHB$ are vertical angles, they are congruent. Therefore, $\triangle CHE \cong \triangle FHB$ by the ASA Postulate.

Exercises

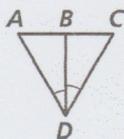
Indicate congruences.

- Copy the top figure at the right. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the ASA Postulate.
- Copy the second figure shown. Mark the figure with the angle congruence and side congruence symbols that you would need to prove the triangles congruent by the AAS Theorem.
- Draw two triangles that are congruent by either the ASA Postulate or the AAS Theorem.

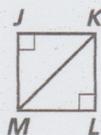


What additional information would you need to prove each pair of triangles congruent by the stated postulate or theorem?

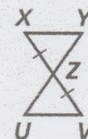
4. ASA



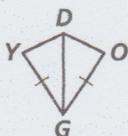
5. AAS



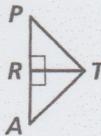
6. SAS



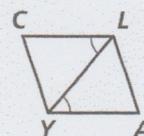
7. SSS



8. AAS



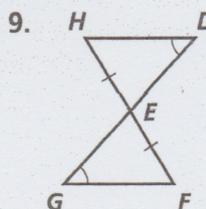
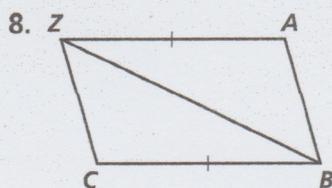
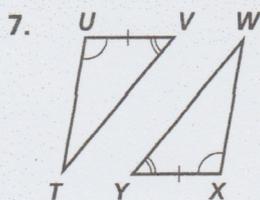
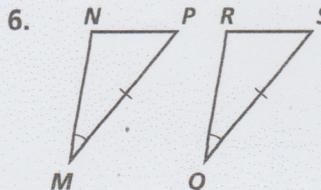
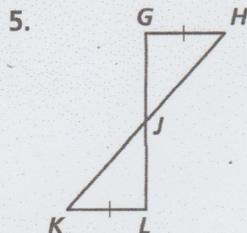
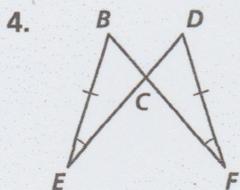
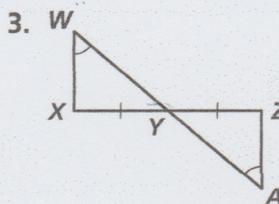
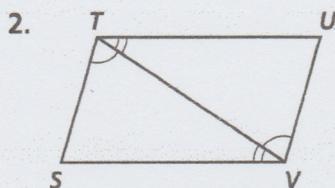
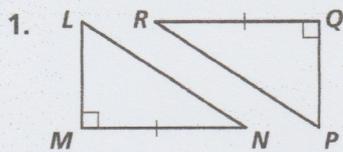
9. ASA



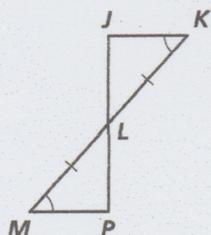
Practice 4-3

Triangle Congruence by ASA and AAS

Tell whether the ASA Postulate or the AAS Theorem can be applied directly to prove the triangles congruent. If the triangles cannot be proved congruent, write *not possible*.



10. Write a two-column proof.
 Given: $\angle K \cong \angle M$, $\overline{KL} \cong \overline{ML}$
 Prove: $\triangle JKL \cong \triangle PML$

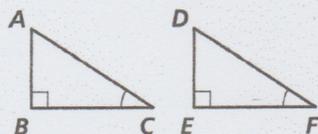


11. Write a flow proof.
 Given: $\angle Q \cong \angle S$, $\angle TRS \cong \angle RTQ$
 Prove: $\triangle QRT \cong \triangle STR$

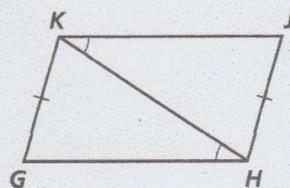


What else must you know to prove the triangles congruent for the reason shown?

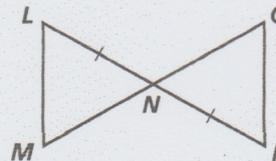
12. ASA



13. AAS



14. ASA



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Reteaching 4-4

Using Congruent Triangles: CPCTC

OBJECTIVE: Using triangle congruence and CPCTC to prove that the parts of two triangles are congruent

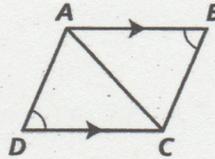
MATERIALS: None

Example

Write a two-column proof.

Given: $\overline{AB} \parallel \overline{DC}$, $\angle B \cong \angle D$

Prove: $\overline{BC} \cong \overline{DA}$



Statements

1. $\overline{AB} \parallel \overline{DC}$
2. $\angle BAC \cong \angle DCA$
3. $\angle B \cong \angle D$
4. $\overline{AC} \cong \overline{AC}$
5. $\triangle ABC \cong \triangle CDA$
6. $\overline{BC} \cong \overline{DA}$

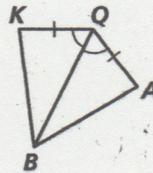
Reasons

1. Given
2. If \parallel lines, then alternate interior \angle s are \cong .
3. Given
4. Reflexive Property of \cong
5. AAS Theorem
6. CPCTC

Exercises

Complete the two-column proof.

1. Given: $\overline{QK} \cong \overline{QA}$; \overline{QB} bisects $\angle KQA$
Prove: $\overline{KB} \cong \overline{AB}$



Statements

- a. ?
2. $\angle KQB \cong \angle AQB$
- c. ?
4. $\triangle KBQ \cong \triangle ABQ$
5. $\overline{KB} \cong \overline{AB}$

Reasons

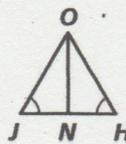
1. Given
- b. ?
3. Reflexive Property of \cong
- d. ?
- e. ?

Write a two-column proof.

2. Given: $\overline{MN} \cong \overline{MP}$, $\overline{NO} \cong \overline{PO}$
Prove: $\angle N \cong \angle P$



3. Given: \overline{ON} bisects $\angle JOH$, $\angle J \cong \angle H$
Prove: $\overline{JN} \cong \overline{HN}$

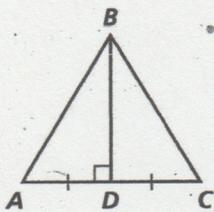


Practice 4-4

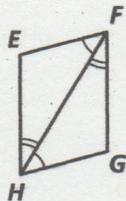
Using Congruent Triangles: CPCTC

Explain how you can use SSS, SAS, ASA, or AAS with CPCTC to prove each statement true.

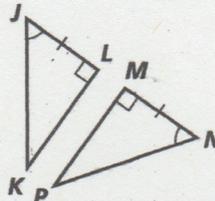
1. $\angle A \cong \angle C$



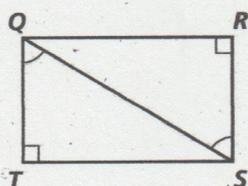
2. $\overline{HE} \cong \overline{FG}$



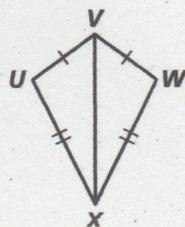
3. $\angle K \cong \angle P$



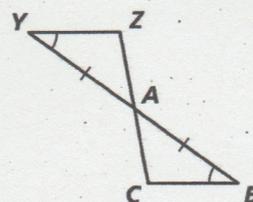
4. $\angle QST \cong \angle SQR$



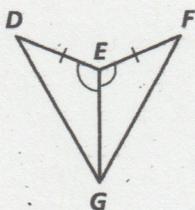
5. $\angle U \cong \angle W$



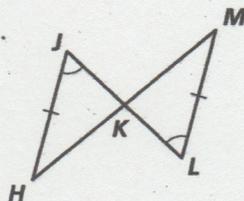
6. $\overline{ZA} \cong \overline{AC}$



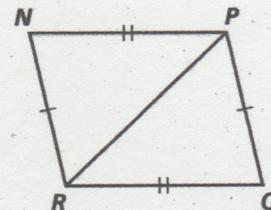
7. $\overline{FG} \cong \overline{DG}$



8. $\overline{JK} \cong \overline{KL}$

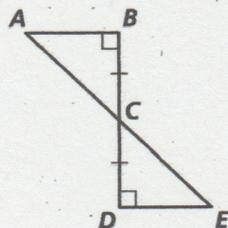


9. $\angle N \cong \angle Q$

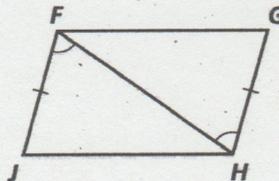


Write a Plan for Proof.

10. Given: $\overline{BD} \perp \overline{AB}$, $\overline{BD} \perp \overline{DE}$, $\overline{BC} \cong \overline{CD}$
 Prove: $\angle A \cong \angle E$



11. Given: $\overline{FJ} \cong \overline{GH}$, $\angle JFH \cong \angle GHF$
 Prove: $\overline{FG} \cong \overline{JH}$



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Reteaching 4-5

Isosceles and Equilateral Triangles

OBJECTIVE: Using and applying properties of isosceles triangles

MATERIALS: None

Example

Find $m\angle ABE$.

Because $AE \cong BE$, $m\angle EAB \cong m\angle ABE$.

$$m\angle EAB + m\angle ABE + m\angle AEB = 180$$

$$m\angle EAB + m\angle ABE + 40 = 180$$

$$m\angle EAB + m\angle ABE = 140$$

$$2m\angle ABE = 140$$

$$m\angle ABE = 70$$

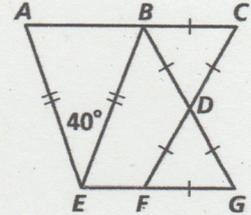
Triangle Angle-Sum Theorem

Substitution

Subtraction Property of Equality

Substitution

Division Property of Equality



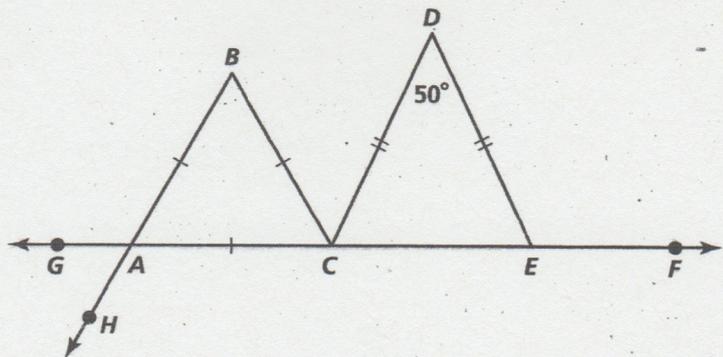
Exercises

Work with a partner to find the measures of the angles of quadrilateral $BDFE$ in the diagram above.

- Find the measures of the angles of $\triangle CBD$ and $\triangle FDG$.
- Use the Angle Addition Postulate to find $m\angle BDF$.
- Use the Angle Addition Postulate to find $m\angle EFC$.
- Use the Angle Addition Postulate to find $m\angle EBG$.
- Use the Polygon Interior Angle-Sum Theorem to find $m\angle BEF$.

Find the measure of each angle.

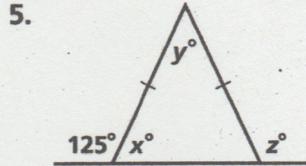
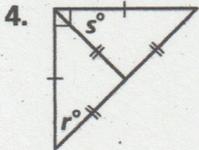
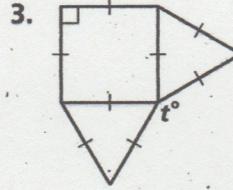
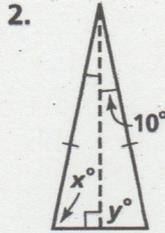
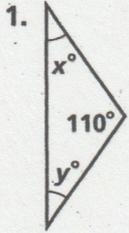
- $m\angle BCA$
- $m\angle DCE$
- $m\angle DEF$
- $m\angle BCD$
- $m\angle BAG$
- $m\angle GAH$



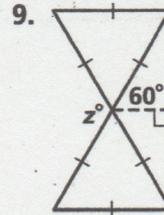
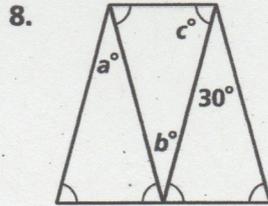
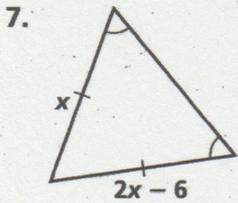
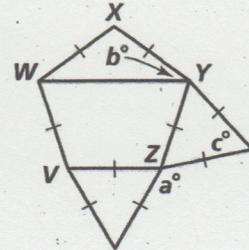
Practice 4-5

Isosceles and Equilateral Triangles

Find the values of the variables.

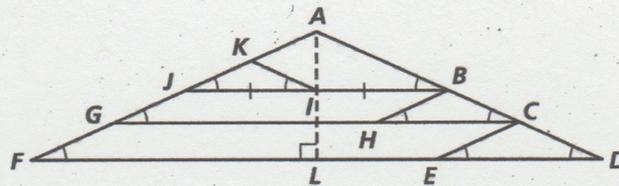


6. WXYZV is a regular polygon.



Complete each statement. Explain why it is true.

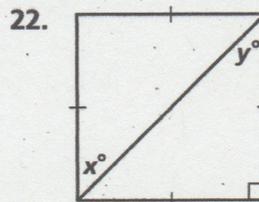
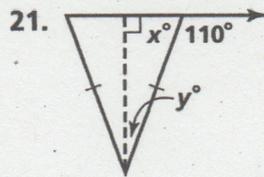
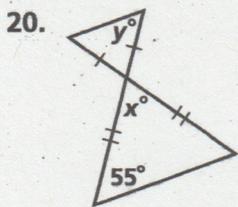
10. $\overline{AF} \cong ?$
11. $\overline{CA} \cong ?$
12. $\overline{KI} \cong ?$
13. $\overline{EC} \cong ?$
14. $\overline{JA} \cong ?$
15. $\overline{HB} \cong ?$



Given $m\angle D = 25$, find the measure of each angle.

16. $\angle JAB$
17. $\angle FAL$
18. $\angle JKI$
19. $\angle DLA$

Find the values of x and y .



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Reteaching 4-6

Congruence in Right Triangles

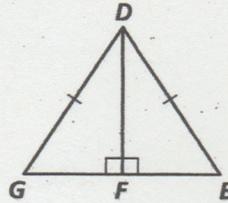
OBJECTIVE: Proving triangles congruent by the HL Theorem

MATERIALS: Ruler

Example

Explain why $\triangle GFD \cong \triangle EFD$ by the HL Theorem.

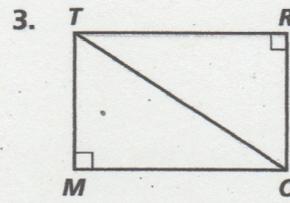
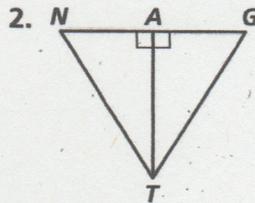
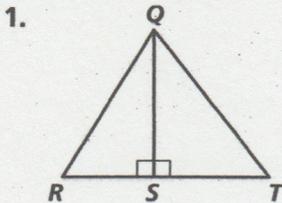
To prove two triangles congruent by the HL Theorem, prove that:



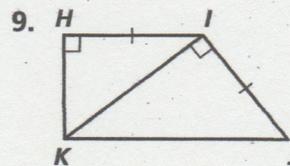
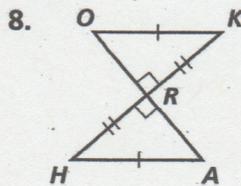
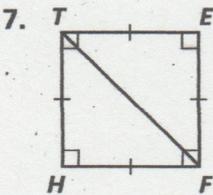
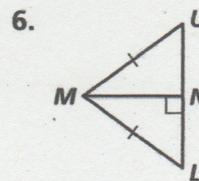
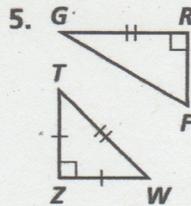
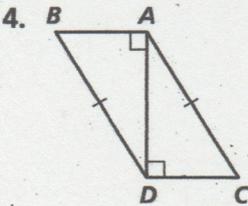
1. They are right triangles.
 2. Their hypotenuses are congruent.
 3. One pair of legs is \cong .
1. $\angle GFD$ and $\angle EFD$ are right angles. Therefore, $\triangle GFD$ and $\triangle EFD$ are right triangles.
 2. $\overline{GD} \cong \overline{ED}$ is given.
 3. $\overline{DF} \cong \overline{DF}$ by the Reflexive Property of \cong .

Exercises

Measure the hypotenuses and the length of one pair of legs to decide whether the triangles are congruent by the HL Theorem. If the triangles are congruent, state the congruence.



Tell whether the HL Theorem can be applied to prove the triangles congruent. If possible, write the triangle congruence.

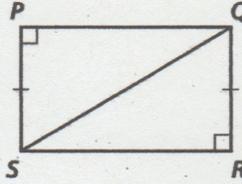
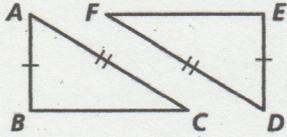


Practice 4-6

Congruence in Right Triangles

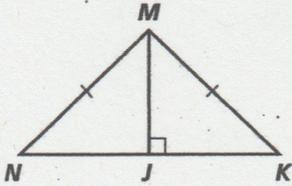
Write a two-column proof.

1. Given: $\overline{AB} \perp \overline{BC}$, $\overline{ED} \perp \overline{FE}$, $\overline{AB} \cong \overline{ED}$, $\overline{AC} \cong \overline{FD}$ 2. Given: $\angle P$ and $\angle R$ are right angles, $\overline{PS} \cong \overline{QR}$
 Prove: $\triangle ABC \cong \triangle DEF$ Prove: $\triangle PQS \cong \triangle RSQ$

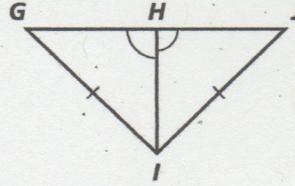


Write a flow proof.

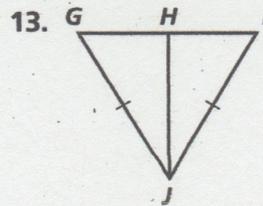
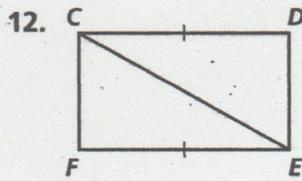
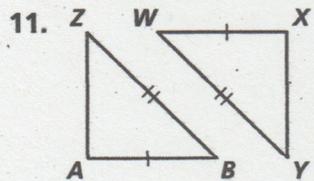
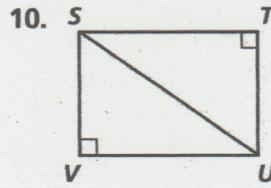
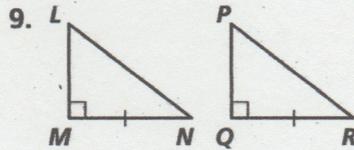
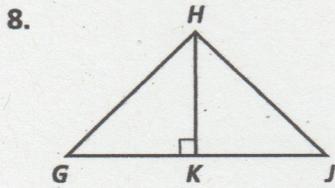
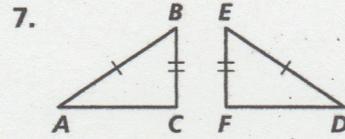
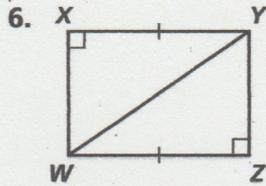
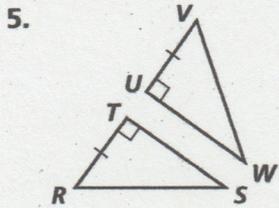
3. Given: $\overline{MJ} \perp \overline{NK}$, $\overline{MN} \cong \overline{MK}$
 Prove: $\triangle MJN \cong \triangle MJK$



4. Given: $\overline{GI} \cong \overline{JI}$, $\angle GHI \cong \angle JHI$
 Prove: $\triangle IHG \cong \triangle IJH$



What additional information do you need to prove each pair of triangles congruent by the HL Theorem?



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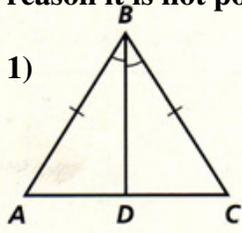
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Geometry
Worksheet – Congruent Triangles

NAME _____

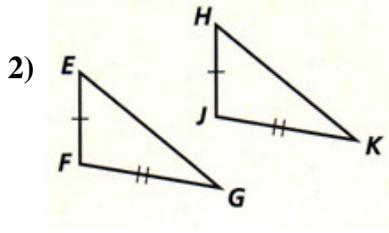
Date _____ HR _____

- a) Determine whether the following triangles are congruent.
 b) If they are, name the triangle congruence (pay attention to proper correspondence when naming the triangles) and then identify the Theorem or Postulate (SSS, SAS, ASA, AAS, HL) that supports your conclusion.
 c) Be sure to show any additional congruence markings you used in your reasoning.
 d) If the triangles cannot be proven congruent, state “not possible.” Then given the reason it is not possible.



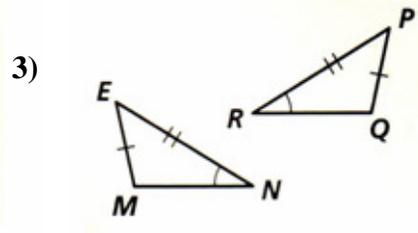
Congruence:
 $\triangle ABD \cong \triangle$ _____

Reason:



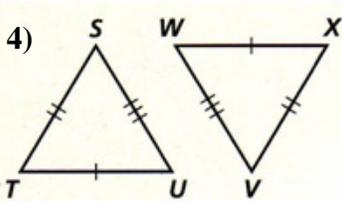
Congruence:
 $\triangle EFG \cong \triangle$ _____

Reason:



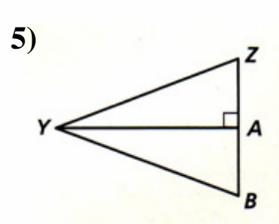
Congruence:
 $\triangle EMN \cong \triangle$ _____

Reason:



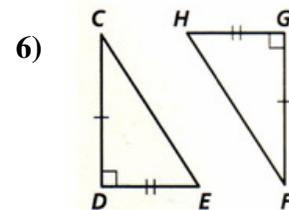
Congruence:
 $\triangle STU \cong \triangle$ _____

Reason:



Congruence:
 $\triangle YZA \cong \triangle$ _____

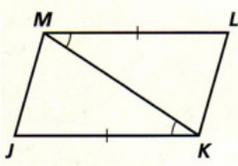
Reason:



Congruence:
 $\triangle CDE \cong \triangle$ _____

Reason:

7)

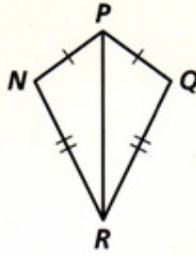


Congruence:

$\Delta KJM \cong \Delta$ _____

Reason:

8)

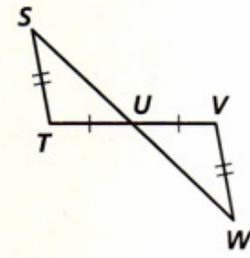


Congruence:

$\Delta NPR \cong \Delta$ _____

Reason:

9)

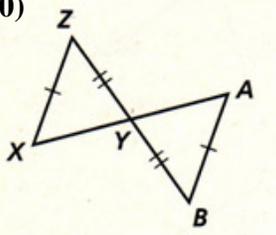


Congruence:

$\Delta STU \cong \Delta$ _____

Reason:

10)

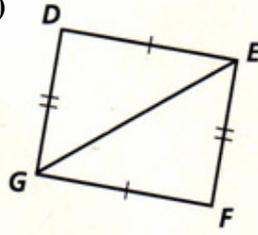


Congruence:

$\Delta XYZ \cong \Delta$ _____

Reason:

11)

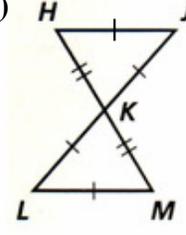


Congruence:

$\Delta DEG \cong \Delta$ _____

Reason:

12)

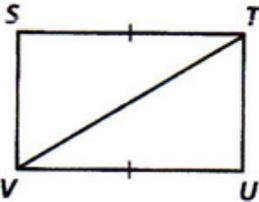


Congruence:

$\Delta HJK \cong \Delta$ _____

Reason:

13)

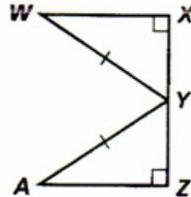


Congruence:

$\Delta STV \cong \Delta$ _____

Reason:

14)

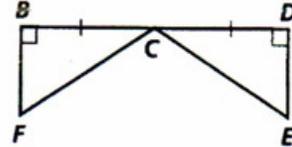


Congruence:

$\Delta WXY \cong \Delta$ _____

Reason:

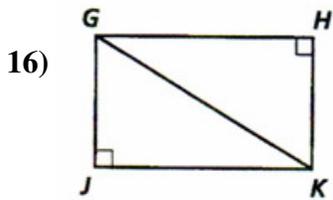
15)



Congruence:

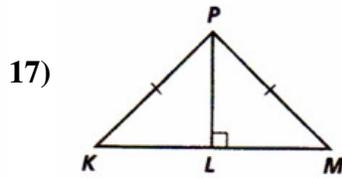
$\Delta BCF \cong \Delta$ _____

Reason:



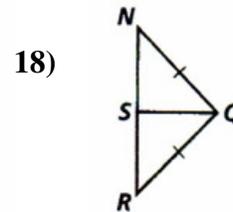
Congruence:
 $\Delta GJK \cong \Delta$ _____

Reason:



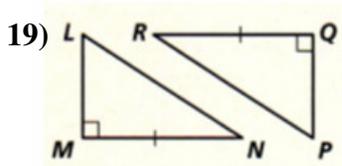
Congruence:
 $\Delta KLP \cong \Delta$ _____

Reason:



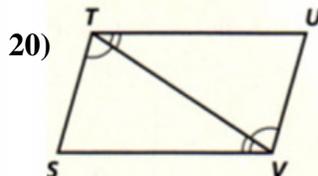
Congruence:
 $\Delta NSQ \cong \Delta$ _____

Reason:



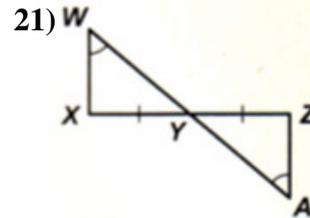
Congruence:
 $\Delta LMN \cong \Delta$ _____

Reason:



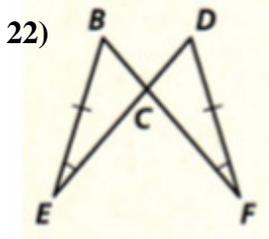
Congruence:
 $\Delta STV \cong \Delta$ _____

Reason:



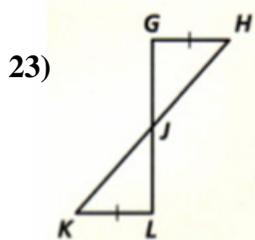
Congruence:
 $\Delta WXY \cong \Delta$ _____

Reason:



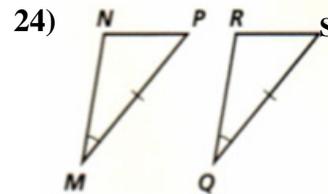
Congruence:
 $\Delta BCE \cong \Delta$ _____

Reason:



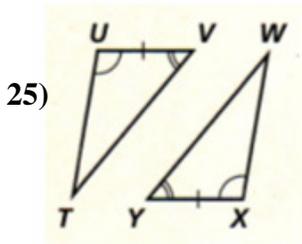
Congruence:
 $\Delta GHJ \cong \Delta$ _____

Reason:



Congruence:
 $\Delta NPM \cong \Delta$ _____

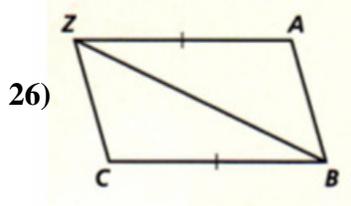
Reason:



Congruence:

$\Delta TUV \cong \Delta$ _____

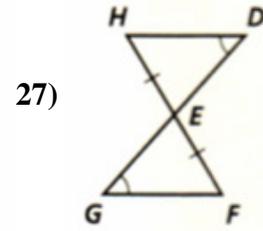
Reason:



Congruence:

$\Delta BCZ \cong \Delta$ _____

Reason:

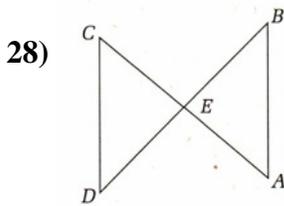


Congruence:

$\Delta EFG \cong \Delta$ _____

Reason:

Use the given information to mark the diagram appropriately. Name the triangle congruence (pay attention to proper correspondence when naming the triangles) and then identify the Theorem or Postulate (SSS, SAS, ASA, AAS, HL) that would be used to prove the triangles congruent. If the triangles cannot be proven congruent, state “not possible.”

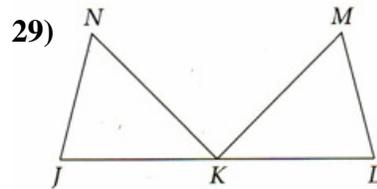


Given: $\overline{CD} \cong \overline{AB}$; $\angle B \cong \angle D$

Congruence:

$\Delta CDE \cong \Delta$ _____

Reason:

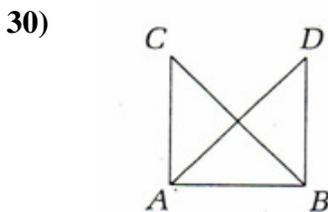


Given: $\overline{JN} \cong \overline{LM}$; $\overline{NK} \cong \overline{MK}$;
 $\angle N \cong \angle M$

Congruence:

$\Delta JKN \cong \Delta$ _____

Reason:

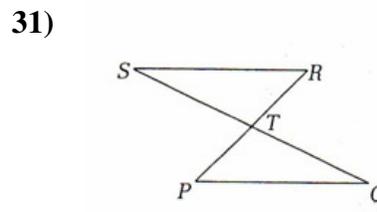


Given: $\overline{AC} \cong \overline{BD}$; $\overline{AD} \cong \overline{BC}$

Congruence:

$\Delta ABC \cong \Delta$ _____

Reason:



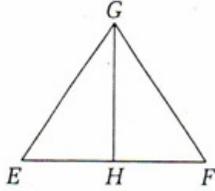
Given: \overline{SQ} and \overline{PR} bisect each other

Congruence:

$\Delta RST \cong \Delta$ _____

Reason:

32)



Given: \overline{GH} bisects $\angle EGF$;
 $\overline{EG} \cong \overline{FG}$

Congruence: $\triangle EGH \cong \triangle$ _____

Reason:

Now choose one of the problems from 28-32 and create a flow chart proof. Then transform your flow chart proof into a 2 column proof. Your “given” will be the “Given” from the problem and your “prove” will be the “Congruence” statement you created.